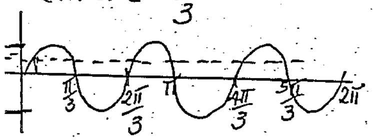
Answers
Modulus 3.3
Trigonometry
"Sunshine"

1) $3\sin 3x = 2 \quad 0 \le x \le 2\pi$

$\sin 3x = \frac{2}{3}$



$3x = \sin^{-1}(2/3)$
$= 0.7297$

$x = 0.243$ (3d.p)
$x = 0.804$
$x = 2.338$
$x = 2.898$
$x = 4.432$
$x = 4.993$

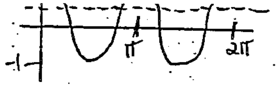
These solutions can be found by drawing $y = \sin 3x$ and $y = 2/3$ on your calculator. Then you solve ISCT and right arrow button to get all solutions.

$3x = 0.7297$
general solution for sine
$3x = n\pi + (-1)^n (0.7297)$
$x = n\pi/3 + (-1)^n (0.243)$

0	$x = 0.243$
1	$x = 0.804$
2	$x = 2.338$

etc.
stop when $x > 2\pi$

1(b) $2\cos 2x = 1$
$\cos 2x = \frac{1}{2}$



$0 \le x \le 2\pi$

$2x = \cos^{-1}(\frac{1}{2})$
$= \frac{\pi}{3}$
$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

or $2x = \pi/3$
General Solution $2x = 2n\pi \pm \frac{\pi}{3}$
$x = n\pi \pm \frac{\pi}{6}$

$n=0 \quad x = \frac{\pi}{6}$
$n=1 \quad x = \frac{5\pi}{6} \quad x = \frac{7\pi}{6}$ etc.

(c) $2\tan 3(x-1) = 4 \quad 0 \le x \le 2\pi$
$\tan 3(x-1) = 2$
$3(x-1) = \tan^{-1}(2)$
$= 1.107$

Using general solution method
$3(x-1) = n\pi + 1.107$
$x - 1 = n\pi/3 + 0.369$
$x = n\pi/3 + 1.369$

$n=0$	$x = 1.369$	$n = -1$, $x = 0.323$
$n=1$	$x = 2.416$	
$n=2$	$x = 3.463$	
$n=3$	$x = 4.511$	
$n=4$	$x = 5.558$	
$n=5$	$x = 6.605$ ← not a solution outside domain for x.	

2. $t = 75\sin\frac{\pi(d-70)}{180} + 430$

$d \sim$ days after 1 Jan 2009

(a) Minimum value of $t = 355$
Maximum value of $t = 505$.

(b) $75\sin\frac{\pi(d-70)}{180} + 430 = 355$
$\sin\frac{\pi(d-70)}{180} = \frac{(355-430)}{75}$
$\frac{\pi(d-70)}{180} = \sin^{-1}(-1)$
$\frac{\pi(d-70)}{180} = 3\pi/2$
$2d - 140 = 540$
$2d = 680$
$d = 340$

$75\sin\frac{\pi(d-70)}{180} + 430 = 505$
$\sin\frac{\pi(d-70)}{180} = \frac{(505-430)}{75}$
$\frac{\pi(d-70)}{180} = \sin^{-1}(1)$
$\frac{\pi(d-70)}{180} = \pi/2$
$2d - 140 = 180$
$2d = 320$
$d = 160$

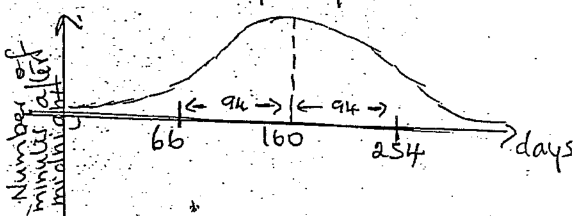
(a) Earliest sun rises is 355 minutes after midnight ie at 5.55am.
Latest sun rises is at 505 minutes after midnight ie at 8.25am
∴ Minutes between these two times = 150 minutes

(b) Days between earliest and latest sunrises
$= 180$

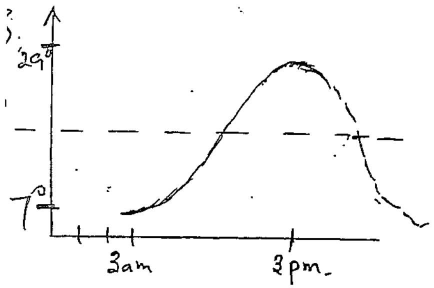
(c) $t = 75\sin\frac{\pi(25-70)}{180} + 430$
$= 376.967$ minutes after midnight
ie. 6.17 am (to nearest minute)

(d) At 7.05 am $\quad t = 425$ (minutes after midnight.)

$75\sin\left(\frac{\pi(d-70)}{180}\right) + 430 = 425.$
$\frac{\pi(d-70)}{180} = \sin^{-1}\left(\frac{425-430}{75}\right)$
$= -0.0667$
$\pi(d-70) = 180 \times -0.0667$
$d - 70 = \frac{180 \times -0.667}{\pi}$
$d = \frac{180 \times -0.667}{\pi} + 70$
$d = 66.177$
ie. 66 days after 1 Jan 2009



∴ Therefore 2nd day in the year when sunrises at 7.05 am is 254 days (to nearest day) after 1 Jan 2009.



15 hours after midnight

$11 \sin \frac{2\pi}{24}(15-M)+18=29$

$\sin \frac{\pi}{12}(15-M) = 1$

$\frac{\pi}{12}(15-M) = \sin^{-1}(1)$

$\frac{\pi}{12}(15-M) = \frac{\pi}{2}$

$15-M = 6$

$\therefore M=9$

Equation of situation is

$y = 11 \sin \pi/12 (x-9)+18$

$x=3$ (ie 3 hours after midnight)

$y = 11 \sin \frac{\pi}{12}(3-9)+18=7$

At 3am temperature is 7^0

At midnight $x=0$

$y = 11 \sin \pi/12 (0-9)+18 = 10.22$

At midnight temperature $= 10.22^0$

At midday $x=12$

$y = 11 \sin \pi/12 (12-9)+18 = 25.78$

$\therefore$ At midday temperature $= 25.78^0$

(e) $11 \sin \frac{\pi}{12}(x-9)+18 = 20$

$\frac{\pi}{12}(x-9) = \sin^{-1}\left(\frac{20-18}{11}\right)$

$= 0.1828$

$x = \frac{0.1828 \times 12}{\pi} + 9$

$= 9.698$

$\therefore$ Temperature is 20^0

9.698 hours after midnight

i.e at 9.42 am (to nearest minute)

4 $L.H.S = \frac{\cos x - \cos 3x}{\sin 3x - \sin x}$

$= \frac{-2\sin 2x \sin(-x)}{2\cos 2x \sin x}$

$\sin(-x) = -\sin x$

$= \frac{2\sin 2x \sin x}{2\cos 2x \sin x}$

$= \tan 2x$

5. $\sin 3(x-\pi) = 0.8$

$3(x-\pi) = \sin^{-1} 0.8$

$= 0.927$

$3(x-\pi) = n\pi + (-1)^n (0.927)$

$x - \pi = \frac{n\pi}{3} + (-1)^n (0.309)$

$x = \frac{n\pi}{3} + \pi + (-1)^n (0.309)$

6

$\sin 4x - \cos x = 0$

See later for an answer to this question

7. $L.H.S = \frac{1+\cos A}{1-\operatorname{cosec} A} + \frac{1-\cos A}{1+\operatorname{cosec} A}$

$= \frac{(1+\cos A)(1+\operatorname{cosec} A) + (1-\cos A)(1-\operatorname{cosec} A)}{(1-\operatorname{cosec} A)(1+\operatorname{cosec} A)}$

$= \frac{1 + \cancel{\cos A} + \cancel{\operatorname{cosec} A} + \cos A \operatorname{cosec} A + 1 - \cancel{\cos A} - \cancel{\operatorname{cosec} A} + \cos A \operatorname{cosec} A}{1 - \operatorname{cosec}^2 A}$

$= \frac{2 + 2\cos A \operatorname{cosec} A}{1 - \operatorname{cosec}^2 A}$

$= \frac{2 + 2\frac{\cos A}{\sin A}}{1 - \cot^2 A - 1} =$

$= \frac{-2}{\cot^2 A} - \frac{2\cot A}{\cot^2 A}$

$= -2\tan^2 A - 2\tan A$

$= -2\tan A(\tan A + 1)$

$= R.H.S$

ANSWERS

"The River Mouth"

1(a) $0 \le x \le 2\pi$

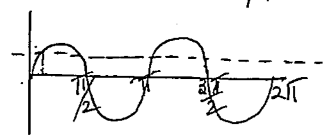
$10 \sin 2x = 5$

$\sin 2x = \frac{1}{2}$

$2x = \sin^{-1}(\frac{1}{2})$

$= \pi/6$

$x = \pi/12$



$x = \pi/12$ $\quad$ $x = \pi/2 - \pi/12 = 5\pi/12$

$x = \pi + \pi/12 = 13\pi/12$ $\quad$ $x = \frac{3\pi}{2} - \frac{\pi}{12}$

$= 17\pi/12$

(b) $4\cos \frac{3x}{2} = 1$

$\cos \frac{3x}{2} = \frac{1}{4}$

$\frac{3x}{2} = \cos^{-1}(\frac{1}{4})$

$= 1.318$

Gen Soln: $\frac{3x}{2} = 2n\pi \pm 1.318$

$x = \frac{4n\pi}{3} \pm \frac{1.318 \times 2}{3}$

$\therefore n=0$ $\quad x = 0.8787$

$n=1$ $\quad x =$ [illegible] 3.310

$x =$ [illegible] 5.067

$n=2$ $\quad x =$ [illegible]

$\tan 2(x+1) = 2$

$(x+1) = \tan^{-1}(2)$

$2(x+1) = 1.107$

$x+1 = 0.554$

$x = -0.446$ (outside range)

aph $y = \tan 2(x+1)$

nd $y = 2$.

$= 1.124$

$= 2.695$

$= 4.255$

$= 5.836$

$C(t) = 2 - 2\cos\left(\frac{\pi}{3}t\right)$

where $2 \le t \le 5$

Max value will be when $\cos \frac{\pi t}{3} = -1$

$\frac{\pi t}{3} = \cos^{-1}(-1)$

$\pi t/3 = \pi$

$t/3 = 1$

$t = 3$

After 3 hours

$C(t) = 2 - 2(-1) = 4$

When $t = 5$

$C(t) = 2 - 2\cos \frac{\pi \times 5}{3} = 1$

$C(t) = 2$

$2 - 2\cos \frac{\pi t}{3} = 2$

$-2\cos \pi t/3 = 0$

$-\cos \frac{\pi t}{3} = 0$

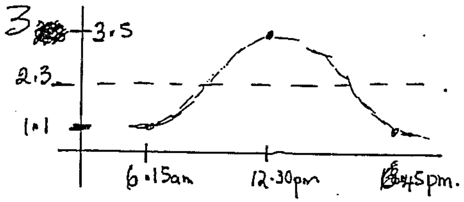
$\therefore \frac{\pi t}{3} = \cos^{-1}(0)$

$\frac{\pi t}{3} = \pi/2$

$t = 1.5$

$\therefore$ After 1.5 hours concentration will be half of maximum.

3



$y = 1.2 \sin \frac{2\pi}{12.5}(x \quad) + 2.3$

At 12.30pm

$1.2 \sin \frac{2\pi}{12.5}(12.5 - M) + 2.3 = 3.5$

hours after midnight.

$\sin \frac{2\pi}{12.5}(12.5 - m) = 1$

$\frac{2\pi}{12.5}(12.5 - m) = \sin^{-1}(1)$

$= \frac{\pi}{2}$

$2\pi - \frac{2\pi m}{12.5} = \frac{\pi}{2}$

$\frac{2\pi m}{12.5} = \frac{3\pi}{2}$

$m = \frac{3 \times 12.5}{4} = 9.375$

$\therefore$ Equation to model this situation is

$y = 1.2 \sin \frac{2\pi}{12.5}(X - 9.375) + 2.3$

Depth of water at mid-day $X = 12$ ie 12 hours after midnight.

3 (cont'd)

$y = 1.2 \sin \frac{2\pi}{12.5}(12 - 9.375) + 2.3$

$= 3.462$

At midday water 3.462 metres above sea bed.

(c) Two hours after high tide is 14.5 hours after midnight.

$y = 1.2 \sin \frac{2\pi}{12.5}(14.5 - 9.375) + 2.3$

$= 2.943$

At 2.30pm water 2.943 metres above sea bed.

(d) $1.2 \sin \frac{2\pi}{12.5}(X - 9.375) + 2.3 = 1.5$

$\sin \frac{2\pi}{12.5}(X - 9.375) = \left(\frac{1.5 - 2.3}{1.2}\right)$

$\frac{2\pi}{12.5}(X - 9.375) = \sin^{-1}\left(-\frac{2}{3}\right)$

$= -0.73$

$X - 9.375 = \frac{-0.73 \times 12.5}{2\pi}$

$X = 7.55$ hours after midnight ie at 7.33 am (to nearest minute)

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4. $5 \sin\left(4x - \frac{\pi}{6}\right) + 3 = 2$

$\sin\left(4x - \frac{\pi}{6}\right) = -\frac{1}{5}$

$4x - \frac{\pi}{6} = \sin^{-1}\left(-\frac{1}{5}\right)$

$= -0.201$

General Solution.

$4x - \frac{\pi}{6} = n\pi + (-1)^n(-0.201)$

$4x = n\pi + (-1)^n(-0.201) + \frac{\pi}{6}$

$x = \frac{n\pi}{4} + (-1)^n(-0.05) + \frac{\pi}{24}$

5.(a) L.H.S $= \sin 3x$,

$= \sin(2x + x)$

$= \sin 2x \cos x + \cos 2x \sin x$

$= 2\sin x \cos x \cos x + (1 - 2\sin^2 x)\sin x$

$= 2\sin x \cos^2 x + \sin x - 2\sin^3 x$

$= 2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x$

$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$= 3\sin x - 4\sin^3 x$

$=$ R.H.S.

5(b) L.H.S $= \frac{\sin 2\alpha - \sin\alpha}{\cos 2\alpha - \cos\alpha + 1}$

$= \frac{2\sin\alpha\cos\alpha - \sin\alpha}{2\cos^2\alpha - 1 - \cos\alpha + 1}$

$= \frac{\sin\alpha(2\cos\alpha - 1)}{\cos\alpha(2\cos\alpha - 1)}$

$= \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$

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6. Do your own drawing!!!

INSWER
QUESTION 6.
Sunshine"

$\text{in } 4x - \cos x = 0.$

$\text{in } 2x \cos 2x - \cos x = 0.$

$\cos x \cos 2x - \cos x = 0$

$x(4 \sin x \cos 2x - 1) = 0$

$x(4 \sin x (1 - 2\sin^2 x) - 1) = 0$

$x(4 \sin x - 8 \sin^3 x - 1) = 0$

$x(8 \sin^3 x - 4 \sin x + 1) = 0.$

et $\sin x = t$

$8t^3 - 4t + 1 = 0$

et $t = \frac{1}{2}$

$8(\frac{1}{2})^3 - 4(\frac{1}{2}) + 1$

$1 - 2 + 1 = 0.$

$t - \frac{1}{2}$ or $2t - 1$ is factor.

$$\begin{array}{r} 4t^2 + 2t - 1 \\ t-1 \overline{) 8t^3 + 0t^2 - 4t + 1} \\ -(8t^3 - 4t^2) \\ \hline 4t^2 - 4t \\ -(4t^2 - 2t) \\ \hline -2t + 1 \\ -2t + 1 \\ \hline \end{array}$$

$8t^3 - 4t + 1 = (2t - 1)(4t^2 + 2t - 1)$

ing solver on alculator or ormula method

$t = \frac{-2 \pm \sqrt{2^2 - 4(4)(-1)}}{8}$

$\therefore t = \frac{1}{2}, \quad t = 0.309,$

$t = -0.809.$

$\therefore$ General solutions

$-\cos x = 0$

$x = \cos^{-1}(0)$

$x = 2n\pi \pm 0$

$x = 2n\pi$

$\sin x = \frac{1}{2} \qquad x = \sin^{-1}(\frac{1}{2})$

$x = \pi/6$

$x = n\pi + (-1)^n (\frac{\pi}{6})$

$\sin x = 0.309$

$x = \sin^{-1}(0.309$

$x = 0.314$

$x = n\pi + (-1)^n (0.314)$

$\sin x = -0.809$

$x = \sin^{-1}(-0.809)$

$x = -0.942$

$x = n\pi + (-1)^n (-0.942)$

ANSWERS
'LUNGS'

1 (a) $3 \cos 2(x + 90^\circ) = 1$

$0 \leq x \leq 360^\circ$

$\cos 2(x + 90^\circ) = \frac{1}{3}$

Graph

$y = \cos 2(x + 90^\circ)$

$y = \frac{1}{3}$

$x = 54.7^\circ$

$x = 125.3^\circ$

$x = 234.7^\circ$

$x = 305.3^\circ$

(b) $0 \leq x \leq 2\pi$

$2 \tan 3x = \sqrt{2}$

$\tan 3x = \frac{1}{\sqrt{2}}$

$3x = \tan^{-1} \frac{1}{\sqrt{2}}$

Graph:

$y = 2 \tan 3x$

$y = \sqrt{2}$

$x = 0.205$

$x = 1.252$

$x = 3.000$

$x = 3.347$

$x = 4.4$

$x = 5.441$

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1 (c) $2 \sin 4(x - 30^\circ) = 1.6$

$0 \leq x \leq 360^\circ$

$\sin 4(x - 30^\circ) = 0.8$

Graph: $y = \sin(4x - 120^\circ)$

$y = 0.8$

$x = 43.3^\circ, \; x = 61.7^\circ, \; x = 133.3^\circ$

$x = 151.7^\circ, \; x = 223.3^\circ, \; x = 241.7^\circ$

$x = 313.3^\circ, \; x = 331.7$

2, $h = 3 - 2 \cos \frac{\pi t}{6}$

Max height 5 metres above reef

Max height when $\cos \frac{\pi t}{6} = -1$

$\pi t/6 = \cos^{-1}(-1)$

$\frac{\pi t}{6} = \pi \quad \therefore t = 6$

ie. 6 hours after mid-day.

min height 1 m above reef.

Mini height when $\cos \frac{\pi t}{6} = 1$

$\pi t/6 = \cos^{-1}(1)$

$\frac{\pi t}{6} = 0 \; \therefore t = 0$

ie at midday.

(a) Difference in heights $= 5 - 1 = 4$ (metres)

(c) Time between low and high tides is 6 hours.

) $h = 3 - 2\cos\frac{\pi t}{6}$

At $t = 2.5$

i.e $2\frac{1}{2}$ hours after midday.

$= 3 - 2\cos\left(\frac{2.5\times\pi}{6}\right)$

$= 2.482$

at 2.30pm the height of water above reef is 2.482 metres.

)

$2.7 = 3 - 2\cos\frac{\pi t}{6}$

$\cos\frac{\pi t}{6} = 0.3$

$\cos\frac{\pi t}{6} = 0.15$

$\frac{\pi t}{6} = \cos^{-1}(0.15)$

$\frac{\pi t}{6} = 1.4202$

$t = \frac{1.4202\times 6}{\pi}$

$t = 2.7124$

Can be refloated 2.7124 hours after midday ie at 2.43pm.

DO NOT DO QUESTION 3. (9)

4. $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-(\sec^2\theta-1)}{\sec^2\theta}$

$= \frac{2-\sec^2\theta}{\sec^2\theta}$

$= \frac{2}{\sec^2\theta} - \frac{\sec^2\theta}{\sec^2\theta}$

$= 2\cos^2\theta - 1 = RHS$

5. $3\sin 5x = 2.16$

$\sin 5x = 0.72$

$5x = \sin^{-1}(0.72)$

$= 0.8038$

General Sol^n

$5x = n\pi + (-1)^n(0.8038)$

$x = \frac{n\pi}{5} + (-1)^n(0.1608)$

6. $h = \sin\frac{\pi t}{2} + \sin\pi t$

$= \sin\pi t + \sin\frac{\pi t}{2}$

$= 2\sin\frac{3\pi t}{4}\cos\frac{\pi t}{4}$

$2\sin\frac{3\pi t}{4}\cos\frac{\pi t}{4} = 0$

when (a) $\sin\frac{3\pi t}{4} = 0$

$\frac{3\pi t}{4} = \sin^{-1}(0)$

$\frac{3\pi t}{4} = 0$

General Solution

$\frac{3\pi t}{4} = n\pi + (-1)^n 0$

$3\pi t = 4n\pi$

$t = \frac{4n}{3}$

when $n=0$ $t=0$

$n=1$ $t = 4/3$ sec.

$n=2$ $t = 8/3 = 2\frac{2}{3}$ sec.

$n=3$ $t = 4$ sec

$n=4$ $t = \frac{16}{3} = 5\frac{1}{3}$ sec.

(b) $\cos\frac{\pi t}{4} = 0$

$\frac{\pi t}{4} = \cos^{-1}(0)$

$\frac{\pi t}{4} = \frac{\pi}{2}$

$\frac{\pi t}{4} = 2n\pi \pm \frac{\pi}{2}$

$\pi t = 8n\pi \pm 2\pi$

$t = \frac{8n\pi}{\pi} \pm \frac{2\pi}{\pi}$

$= 8n \pm 2$

At $n=0$ $t = 2$ sec.

$n=1$ $t = 6$ sec.

7. $\cos 3x\sin x = \cos 4x\sin 2x$

$\cos 3x\sin x = 2\cos 4x\cos x\sin x$

N.B. By dividing thru' by $\sin x$ a series of solutions have been eliminated

(10)

$\cos 3x\sin x = \sin x[2\cos 4x\cos x]$

N.B. From sum formulas

$\frac{c+d}{2} = 4$ $\quad \frac{c-d}{2} = 1$

$c+d = 8$

$c-d = 2$

$2c = 10$

$c = 5$

$d = 3$

$\therefore \cos 3x\sin x = \sin x(\cos 5x + \cos 3x)$

$\sin x(\cos 3x - \cos 5x - \cos 3x) = 0$

$\sin x\cos 5x = 0$

$\therefore \sin x = 0$ or $\cos 5x = 0$

$x = \sin^{-1}(0)$ $\quad x = \frac{2n\pi}{5} \pm \frac{\pi}{10}$

$x = n\pi + (-1)^n(0)$

$x = n\pi$

$$\frac{1-\sin A}{1-\sec A} - \frac{1+\sin A}{1+\sec A}$$

$$= \frac{(1-\sin A)(1+\sec A) - (1+\sin A)(1-\sec A)}{1-\sec^2 A}$$

$$= \frac{\cancel{1} + \sec A - \sin A - \cancel{\sin A \sec A} - \cancel{1} + \sec A - \sin A + \cancel{\sin A \sec A}}{1-\sec^2 A}$$

$$= \frac{2\sec A}{-\tan^2 A} + \frac{2\sin A}{\tan^2 A} = \frac{2\sin A}{\tan^2 A} - \frac{2\sec A}{\tan^2 A}$$

$$= 2\sin A \frac{\cos^2 A}{\sin^2 A} - \frac{2}{\cos A} \times \frac{\cos^2 A}{\sin^2 A}$$

$$= 2\cos A . \frac{\cos A}{\sin A} - \frac{2\cos A}{\sin A} . \frac{1}{\sin A}$$

$$= 2\cot A\left(\cos A - \frac{1}{\sin A}\right)$$

$$= 2\cot A\,(\cos A - \operatorname{cosec} A) = \text{R.H.S.}$$

E.OE. $\blacksquare$