2006

Internal Assessment Resource

Subject Reference: Calculus 3.3

Internal assessment resource reference number Calculus/3/3 – B version 3

The River Mouth

Supports internal assessment for
Achievement standard 90637 v2
Solve problems and equations involving trigonometric functions
Credits: 4

Student Instructions Sheet

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1. Solve each of the following equations for $0 \le x \le 2\pi$

   (a) $10 \sin 2x = 5$ (A2)

   (b) $4 \cos \frac{3x}{2} = 1$ (A2)

   (c) $3 \tan 2(x+1) = 6$ (A2)

2 Sue has been taking some medication.
She visits the doctor who prescribes her more medication.
She takes the first dose two hours after her visit.

A model has been given for the concentration C units of the drug in her bloodstream at time t hours after Sue visits the doctor.
The model is

$$C(t) = 2 - 2\cos\left(\frac{\pi t}{3}\right) \qquad \text{where } 2 \le t \le 5$$

(a) What is the maximum concentration of the drug in her bloodstream? (A1)

(b) What is the concentration of the drug in her bloodstream after the 5 hours? (A1)

(c) How long does it take for the concentration of the drug in her blood stream to fall to half of its maximum concentration? (A1)

(d) To be effective in controlling the symptoms for which it is prescribed, the concentration of the drug in the bloodstream must be kept at a level of at least 1.8 units. Sue took a second dose 4 hours and 20 minutes after taking the first. Was this soon enough? (M2)

3. At Pipi Bay pipis lie just beneath the surface of the sand.
At the end of the Pipi Bay wharf, the height of the tide above the sea bed can be modelled by a trigonometric function.
The maximum depth of water above the sea bed is 3.5 m and the minimum depth is 1.1 m.
High tide is 6.25 hours after low tide. Low tide today is at 6:15 am.

(a) Obtain the equation of the model for the depth of the water at the end of the wharf where d metres is the depth of the water at time t hours after midnight. (M1)

(b) Use your model to find the depth of the water at mid-day. (M2)

(c) Pipis can be gathered when the tide is less than 1.5 m deep at the end of the wharf. What is the earliest time of the day that the pipis could be gathered? (M2)

4 Find the general solution of the equation $5\sin(4x-\frac{\pi}{6})+3=2$ (M3)

5 Prove each of the following identities.

(a) $\sin 3x = 3\sin x - 4\sin^3 x$ (M3)

(b) $\dfrac{\sin 2a - \sin a}{\cos 2a - \cos a + 1} = \tan a$ (M3)

6 There is a reef at the mouth of a river.
A light house is built on the reef the surface of which is at approximately the same height as the river bank.
Search and Rescue needs to know the height of the light house.
The only equipment available is a compass, theodolite or clinometer and a tape measure.
The light house cannot be reached directly from the river bank.
Indicate measurements that could to be taken, and derive a formula for the height of the light house in terms of these measurements. Clearly identify the meaning of all symbols that you use.
The river bank can be assumed to be horizontal. (E)

'The River Mouth' Assessment

1 a) $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$ rad

b) $x = 0.879, 3.310, 5.067$ rad

c) $x = 1.125, 2.695, 4.266, 5.837$ rad

2 a) 4 units

b) 1 unit

c) $t = 4.5$
It takes $2\frac{1}{2}$ hours after her first dose.

d) Concentration first falls to 1.8 units around $t = 4$ hr 36 min, i.e. 2 hours 36 mins after taking the first dose. She took her 2nd dose too late.

3 a) $h = 1.2\cos\left(\frac{4\pi}{25}t\right) + 2.3$

b) $h = 3.462$ m

c) $t = $ 4:35 am

4) $5\sin\left(4x - \frac{\pi}{6}\right) + 3 = 2$

$\sin\left(4x - \frac{\pi}{6}\right) = -0.2$

$4x - \frac{\pi}{6} = -0.201$ rad (α)

G.S: $4x - \frac{\pi}{6} = n\pi + (-1)^n(-0.201)$

$4x = n\pi + \frac{\pi}{6} + (-1)^n(-0.201)$

$x = \frac{n\pi}{4} + \frac{\pi}{24} + (-1)^n(-0.05)$

M3

5 a) Prove $\sin 3x = 3\sin x - 4\sin^3 x$

$$\begin{aligned}
LHS &= \sin 3x \\
&= \sin(x + 2x) \\
&= \sin x \cos 2x + \cos x \sin 2x && \text{[compound angles} \\
&= \sin x(1 - 2\sin^2 x) + \cos x(2\sin x \cos x) && \text{[double angles} \\
&= \sin x - 2\sin^3 x + 2\sin x \cos^2 x \\
&= \sin x - 2\sin^3 x + 2\sin x(1 - \sin^2 x) \\
&= \sin x - 2\sin^3 x + 2\sin x - 2\sin^3 x \\
&= 3\sin x - 4\sin^3 x \\
&= RHS.
\end{aligned}$$

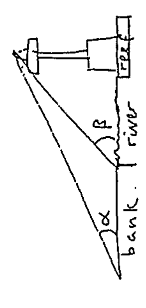
M3

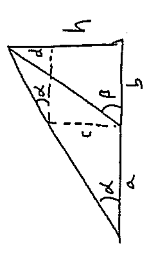
b)
$$\begin{aligned}
LHS &= \frac{\sin 2a - \sin a}{\cos 2a - \cos a + 1} \\
&= \frac{2\sin a \cos a - \sin a}{2\cos^2 a - 1 - \cos a + 1} \\
&= \frac{\sin a(2\cos a - 1)}{2\cos^2 a - \cos a} \\
&= \frac{\sin a\cancel{(2\cos a - 1)}}{\cos a\cancel{(2\cos a - 1)}} \\
&= \tan a \\
&= RHS
\end{aligned}$$

M3

6) Theodolite : instrument for measuring both horizontal and vertical angles.

Clinometer : instrument for measuring elevation angles above horizontal.





$\boxed{h = c + d}$

$\frac{c}{a} = \tan\alpha$; $\boxed{c = a\tan\alpha}$

$\frac{d}{b} = \tan\alpha$; $\boxed{d = b\tan\alpha}$

$\frac{h}{b} = \tan\beta$; $\boxed{b = \frac{h}{\tan\beta}}$

$$\begin{aligned}
h &= c + d \\
h &= a\tan\alpha + b\tan\alpha \\
h &= a\tan\alpha + \left(\frac{h}{\tan\beta}\right)\tan\alpha \\
h - h\frac{\tan\alpha}{\tan\beta} &= a\tan\alpha \\
h\left(1 - \frac{\tan\alpha}{\tan\beta}\right) &= a\tan\alpha \\
h &= \frac{a\tan\alpha}{1 - \frac{\tan\alpha}{\tan\beta}}
\end{aligned}$$

multiply top and bottom by $\tan\beta$

$$h = \frac{a\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}$$ *