8 SIMPLE HARMONIC MOTION

Objectives

After studying this chapter you should

- be able to model oscillations;
- be able to derive laws to describe oscillations;
- be able to use Hooke's Law;
- understand simple harmonic motion.

8.0 Introduction

One of the most common uses of oscillations has been in time-keeping purposes. In many modern clocks quartz is used for this purpose. However traditional clocks have made use of the pendulum. In this next section you will investigate how the motion of a pendulum depends on its physical characteristics.

The key feature of the motion is the time taken for one complete oscillation or swing of the pendulum. i.e. when the pendulum is again travelling in the same direction as the initial motion. The time taken for one complete oscillation is called the **period**.

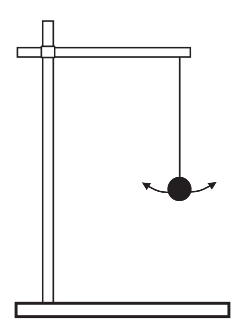
8.1 Pendulum experiments

Activity 1 Your intuitive ideas

To begin your investigation you will need to set up a simple pendulum as shown in the diagram. You will need to be able to

- vary the length of the string;
- vary the mass on the end of the string;
- record the time taken for a particular number of oscillations.

Once you are familiar with the apparatus try to decide which of the factors listed at the beginning of the next page affect the period. Do this without using the apparatus, but giving the answers that you intuitively expect.



The mass is attached by a string to the support, to form a simple pendulum.

(a) The length of the string

(b) The mass of the object on the end of the string.

(c) The initial starting position of the mass.

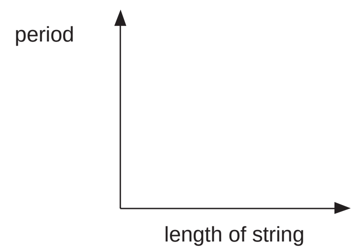
Now try simple experiments to verify or disprove your intuitive ideas, using a table to record your results.

You are now in a position to start analysing the data obtained, using some of the basic mathematical concepts in pure mathematics.

Activity 2 Analysis of results

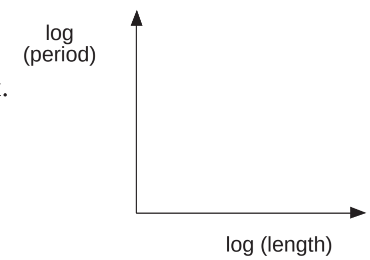
You will probably have observed already that as you shortened the string the period decreased. Now you can begin to investigate further the relationship between the length of the string and the period.

- Plot a graph for your results, showing period against length of string.
- Describe as fully as possible how the period varies with the length of the string.



You may know from your knowledge of pure mathematics how a straight line can be obtained from your results using a log-log plot.

- Plot a graph of log (period) against log (length of string), and draw a line of best fit.
- Find the equation of the line you obtain and hence find the relationship between the period and the length of the string.

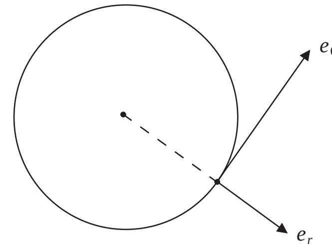


Log-log graph

8.2 Pendulum theory

You will have observed from Section 8.1 that the period of the motion of a simple pendulum is approximately proportional to the square root of the length of the string. In this section you are presented with a theoretical approach to the problem

The path of the mass is clearly an arc of a circle and so the results from Chapter 7, Circular Motion, will be of use here. It is convenient to use the unit vectors, $\mathbf{e}_r$ and $\mathbf{e}_\theta$, directed outwards along the radius and along the tangent respectively. The acceleration, **a**, of an object in circular motion is now given by



The unit vectors for circular motion

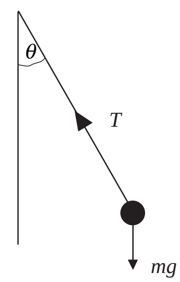
$$\mathbf{a} = -r\dot{\theta}^2\mathbf{e}_r + r\ddot{\theta}\mathbf{e}_\theta$$

where $\dot{\theta} = \frac{d\theta}{dt}$ and $\ddot{\theta} = \frac{d^2\theta}{dt^2}$.

Forces acting on the pendulum

As in all mechanics problems, the first step you must take is to identify the **forces** acting. In this case there is the tension in the string and the force of gravity. There will, of course, also be air resistance, but you should assume that this is negligible in this case. The forces acting and their resultant are summarised in the table below.

Force	Component form
T	$-T\mathbf{e}_r$
mg	$mg\cos\theta\,\mathbf{e}_r - mg\sin\theta\,\mathbf{e}_\theta$
Resultant	$(mg\cos\theta - T)\mathbf{e}_r - mg\sin\theta\,\mathbf{e}_\theta$



Now it is possible to apply Newton's second law, using the expression for the acceleration of an object in circular motion

$$\mathbf{F} = m\mathbf{a},$$

giving

$$(mg\cos\theta - T)\mathbf{e}_r - mg\sin\theta\,\mathbf{e}_\theta = m\left(-l\dot{\theta}^2\,\mathbf{e}_r + l\ddot{\theta}\mathbf{e}_\theta\right)$$

where $r = l,$ the length of the string.

Equating the coefficients of $\mathbf{e}_r$ and $\mathbf{e}_\theta$ in this equation leads to

$$T - mg\cos\theta = ml\dot{\theta}^2 \qquad (1)$$

and

$$-mg\sin\theta = ml\ddot{\theta}. \qquad (2)$$

From equation (2) $\ddot{\theta} = -\dfrac{g\sin\theta}{l}.$

If the size of θ is small, then $\sin\theta$ can be approximated by θ, so that

$$\boxed{\ddot{\theta} = -\frac{g\theta}{l}} \qquad (3)$$

Activity 3 Solving the equation

Verify that

$$\theta = A\cos\left(\sqrt{\frac{g}{l}}\,t + \alpha\right)$$

is a solution of equation (3), where α is an arbitrary constant.

Interpreting the solution

Each part of the solution

$$\theta = A\cos\left(\sqrt{\frac{g}{l}}\,t + \alpha\right)$$

describes some aspect of the motion of the pendulum.

- The variable, *A*, is known as the **amplitude** of the oscillation. In this case the value of *A* is equal to the greatest angle that the string makes with the vertical.

- $\sqrt{\frac{g}{l}}$ determines how long it takes for one complete oscillation.

  When $\sqrt{\frac{g}{l}}\,t = 2\pi$ or $t = 2\pi\sqrt{\frac{l}{g}}$

  then the pendulum has completed its first oscillation. This time is known as the **period** of the motion.

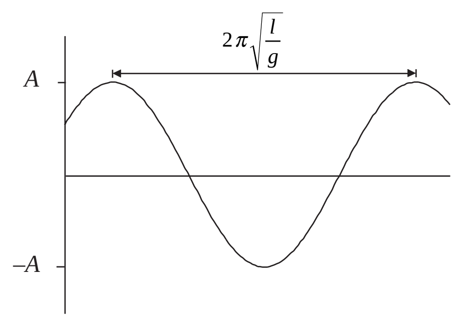
  In general, if you have motion that can be described by

  $$x = A\cos(\omega t + \alpha)$$

  then the period, *P*, is given by

  $$P = \frac{2\pi}{\omega}.$$

  It is sometimes also useful to talk about the **frequency** of an oscillation. This is defined as the number of oscillations per second.



- The constant α is called the **angle of phase**, or simply the **phase**, and its value depends on the way in which the pendulum is set in motion. If it is released from rest the angle of phase will be zero, but if it is flicked in some way, the angle of phase will have a non-zero value.

Exercise 8A

1. A D.I.Y magazine claims that a clock that is fast (i.e. gaining time) can be slowed down by sticking a small lump of Blu-tac to the back of the pendulum. Comment on this procedure.
2. A clock manufacturer wishes to produce a clock, operated by a pendulum. It has been decided that a pendulum of length 15 cm will fit well into an available clock casing. Find the period of this pendulum.
3. The result obtained for the simple pendulum used the fact that $\sin\theta$ is approximately equal to θ for small θ. For what range of values of θ is this a good approximation? How does this affect the pendulum physically?
4. A clock regulated by a pendulum gains 10 minutes every day. How should the pendulum be altered to correct the time-keeping of the clock?
5. Two identical simple pendulums are set into motion. One is released from rest and the other with a push, both from the same initial position. How do the amplitude and period of the subsequent motions compare?
6. The pendulums in Question 5 have strings of length 20 cm and masses of 20 grams. Find equations for their displacement from the vertical if they were initially at an angle of $5°$ to the vertical and the one that was pushed was given an initial velocity of 0.5ms^{-1}.

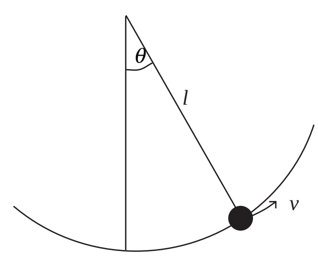
8.3 Energy consideration

An alternative approach is to use energy consideration.

As the simple pendulum moves there is an interchange of kinetic and potential energy. At the extremeties of the swing there is zero kinetic energy and the maximum potential energy. At the lowest point of the swing the bob has its maximum kinetic energy and its minimum potential energy.

Activity 4 Energy

For the simple pendulum shown opposite, find an expression for the height of the pendulum bob in terms of the angle θ. You may assume that you are measuring from the lowest point.



Explain why the potential energy of the bob is given by

$$mgl(1-\cos\theta).$$

The kinetic energy of the pendulum bob is given by $\frac{1}{2}mv^2$.

As no energy is lost from the system, the sum of potential and kinetic energies will always be constant, giving

$$T = mgl(1-\cos\theta) + \frac{1}{2}mv^2$$

where T is the total energy of the system. The value of T can be found by considering the way in which the pendulum is set into motion.

Finding the speed

Solving the equation for v gives

$$v = \sqrt{\frac{2T}{m} - 2gl(1 - \cos\theta)}.$$

This allows you to calculate the speed at any position of the pendulum.

You can also find an expression for θ from the equation for v.

Activity 5 Finding θ

Explain why the speed of the pendulum bob is given by

$$v = l\frac{d\theta}{dt}.$$

(You may need to refer to the Chapter 7, Circular Motion if you find this difficult.)

Substitute this into the equation for v and show that

$$\frac{d\theta}{dt} = \sqrt{\frac{2T}{ml^2} - \frac{2g}{l}(1 - \cos\theta)}.$$

Simplifying the equation

If you assume that the oscillations of the pendulum are small, then you can use an approximation for $\cos\theta$.

Activity 6 Small θ approximation

Use the approximation

$$\cos\theta \approx 1 - \frac{\theta^2}{2}$$

to show that $\frac{d\theta}{dt}$ can be written in the form

$$\frac{d\theta}{dt} = \omega\sqrt{a^2 - \theta^2}, \text{ where } a \text{ is a constant.}$$

Integrating the equation

You can solve this equation by separating the variables to give

$$\int \frac{1}{\sqrt{a^2-\theta^2}}\,d\theta = \int \omega\,dt.$$

The LHS is simply a standard integral that you can find in your tables book and the RHS is the integral of a constant.

Activity 7 Integrating

Show that

$$\theta = \alpha \sin(\omega t + c)$$

and explain why this could be written as

$$\theta = \alpha \cos(\omega t + \alpha)$$

where α and ω are defined as above and c is an unknown constant given by $\alpha = c + \dfrac{\pi}{2}$.

This result is identical to that obtained earlier.

Exercise 8B

1. A pendulum consists of a string of length 30 cm and a bob of mass 50 grams. It is released from rest at an angle of $10°$ to the vertical. Draw graphs to show how its potential and kinetic energy varies with θ. Find the maximum speed of the bob.

2. Find expressions for the maximum speed that can be reached by a pendulum if it is set in motion at an angle $\theta°$ to the vertical if
   (a) it is at rest;
   (b) it has an initial speed u.

8.4 Modelling oscillations

In this section you will investigate other quantities which change with time, can be modelled as oscillations and can be described by an equation in the form $x = a\cos(\omega t + \alpha)$.

There are many other quantities that involve motion that can be described using this equation.

Some examples are the heights of tides, the motion of the needle in a sewing machine and the motion of the pistons in a car's engine. In some cases the motion fits exactly the form given above, but in others it is a good approximation.

Activity 8

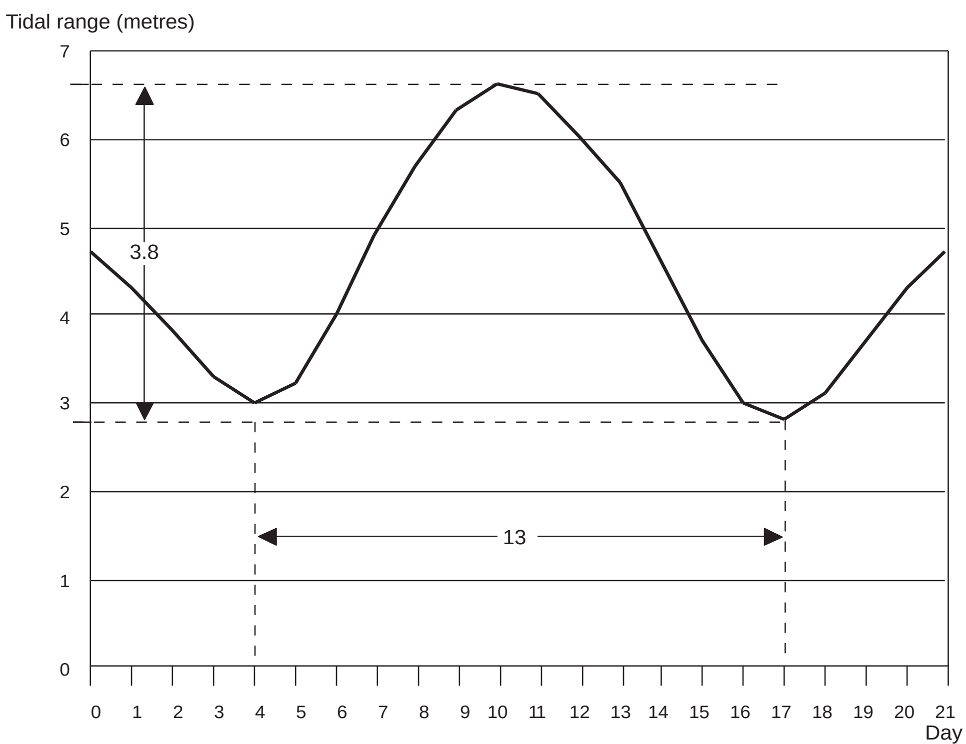
Find other examples of motion that can be modelled using the equation $x = a\cos(\omega t + \alpha)$.

Fitting the equation to data

One example that could be modelled as an oscillation using the equation is the range of a tide (i.e. the difference between high and low tides). The table shows this range for a three-week period.

Day	Range	Day	Range	Day	Range
1	4.3	8	5.7	15	3.7
2	3.8	9	6.3	16	3.0
3	3.3	10	6.6	17	2.8
4	3.0	11	6.5	18	3.1
5	3.2	12	6.0	19	3.7
6	4.0	13	5.5	20	4.3
7	4.9	14	4.6	21	4.7

The graph below shows range against day.



This graph is clearly one that could be modelled fairly well as an oscillation using $x = a\cos(\omega t + \alpha)$. One difference you will observe between this graph and those for the simple pendulum is that this one is not symmetrical about the time axis. The curve has, in fact, been translated upwards, so the range will be described by an equation of the form

$$R = a\cos(\omega t + \alpha) + b.$$

The difference between the maximum and minimum range is approximately 3.8 m. The amplitude of the oscillation will be half this value, 1.9 m. So the value of *a* in the equation will be 1.9.

As the graph is symmetrical about the line $r = 4.7$ the value of *b* will be 4.7.

It is also possible from the graph to see that the period is 13 days. From Section 8.2 you will recall that the period is given by

$$P = \frac{2\pi}{\omega}$$

so that $$\omega = \frac{2\pi}{P}.$$

In this case, $\omega = \dfrac{2\pi}{13}$, which is in radians.

This leaves the value of α to be determined. The equation is now

$$R = 1.9\cos\left(\frac{2\pi t}{13} + \alpha\right) + 4.7.$$

When $t = 1$, $R = 4.3$, so using these values

$$4.3 = 1.9\cos\left(\frac{2\pi}{13} + \alpha\right) + 4.7$$

$$\Rightarrow \qquad 1.9\cos\left(\frac{2\pi}{13} + \alpha\right) = -0.4$$

$$\Rightarrow \qquad \cos\left(\frac{2\pi}{13} + \alpha\right) = -\frac{0.4}{1.9}$$

$$\Rightarrow \qquad \frac{2\pi}{13} + \alpha = 1.78$$

$$\Rightarrow \qquad \alpha = 1.78 - \frac{2\pi}{13}$$

$$\alpha \approx 1.30$$

which gives

$$R = 1.9\cos\left(\frac{2\pi t}{13} + 1.3\right) + 4.7$$

(Note that in applying this result you must use radians.)

Activity 9

Draw your own graph of the data on tidal ranges.

Superimpose on it the graph of the model developed above.

Compare and comment.

Exercise 8C

1. In the UK 240 volt alternating current with a frequency of 50 Hz is utilised. Describe the voltage at an instant of time mathematically.
2. The tip of the needle of a sewing machine moves up and down 2 cm. The maximum speed that it reaches is 4 ms^{-1}. Find an equation to describe its motion.
3. Black and Decker BD538SE jigsaws operate at between 800 and 3200 strokes per minute, the tip of the blade moving 17 mm from the top to the bottom of the stroke. Find the range of maximum speeds for the blade.
4. A person's blood pressure varies between a maximum (systolic) and a minimum (diastolic) pressures. For an average person these pressures are 120 mb and 70 mb respectively. Given that blood pressure can be modelled as an oscillation, find a mathematical model to describe the changes in blood pressure. It takes 1.05 seconds for the blood pressure to complete one oscillation.
5. The motion of the fore and hind wings of a locust can be modelled approximately using the ideas of oscillations. The motion of the fore wings is modelled by

   $$F = 1.5 + 0.5\sin(1.05t - 0.005)$$

   where F is the angle between the fore wing and the vertical. Find the period and the amplitude of the motion.
   Each hind wing initially makes an angle of $1.5°$ to the vertical. It then oscillates with period 0.06 s and amplitude $1.5°$. Form a model of the form

   $$h = H + a\sin(kt)$$

   for the motion of one of the hind wings.

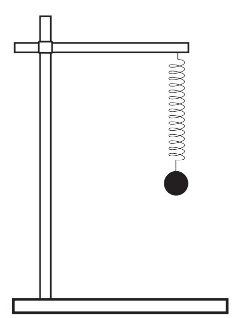
8.5 Springs and oscillations

In Section 6.5, Hooke's Law was used as the model that is universally accepted for describing the relationship between the tension and extension of a spring. Hooke's Law states that

$$T = ke$$

where T is the tension, k the spring stiffness constant and e the extension of the spring. If the force is measured in Newtons and the extension in metres, then k will have units Nm^{-1}.

To begin your investigations of the oscillations of a mass/spring system you will need to set up the apparatus as shown in the diagram opposite.



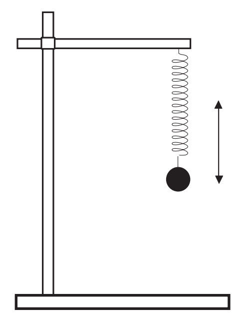
Masses can be attached to the spring suspended from the stand

The equipment you will need is

- a stand to support your springs
- a variety of masses
- 2 or 3 identical springs
- a ruler
- a stopwatch.

Activity 10 Intuitive ideas

If you pull down the mass a little and then release it, it will oscillate, up and down. Once you are familiar with the apparatus, try to decide how the factors listed below affect the period. Do this without using the apparatus, giving the answers that you intuitively expect.



(a) The mass attached to the spring.

(b) The stiffness of the spring.

(c) The initial displacement of the mass.

Also consider some simple experiments to verify or disprove your intuitive ideas.

Theoretical analysis

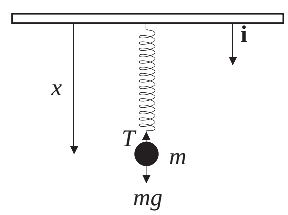
To begin your theoretical analysis you need to identify the resultant force.

Explain why the resultant force is $(mg - T)\mathbf{i}$

Activity 11

Use Hooke's Law to express T in terms of k, l and x, where k is the stiffness of the spring, l the natural length of the spring and x the length of the spring.

So the resultant force on the mass is



$$(mg - kx + kl)\mathbf{i}$$

and the acceleration of the mass will be given by $\frac{d^2x}{dt^2}\mathbf{i}$

Activity 12

Use Newton's second law to obtain the equation

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = g + \frac{kl}{m}$$

and show that

$$x = a\cos\left(\sqrt{\frac{k}{m}}t + \alpha\right) + \frac{mg}{k} + l$$

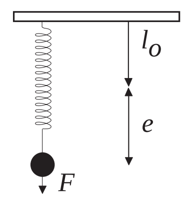
satisfies this equation.

Why is the period of these oscillations given by

$$P = 2\pi\sqrt{\frac{m}{k}} \textbf{ ?}$$

Work done

Consider a spring of stiffness k and natural length l_0. If a force, F, is applied to cause the spring to stretch, then this force must increase as the spring extends. So the **work done** in stretching a spring is evaluated using



Work done in stretching a spring

$$\begin{aligned}\text{Work done} &= \int_0^e F\,dx \\ &= \int_0^e kx\,dx \\ &= \left[\frac{1}{2}kx^2\right]_0^e \\ &= \frac{1}{2}ke^2.\end{aligned}$$

So the energy stored in a spring is given by $\frac{1}{2}ke^2$. When the spring is released the energy is converted into either kinetic or potential energy.

Simple harmonic motion

If the equation describing the motion of an object is of the form

$$x = a\cos(\omega t + \alpha) + c$$

then that type of motion is described as **simple harmonic motion** (SHM).

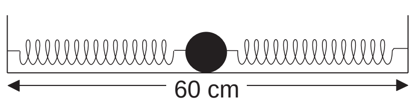
This equation is deduced from the differential equation

$$\frac{d^2x}{dt^2} + \omega^2 x = b$$

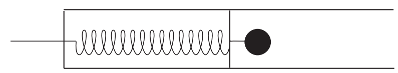
Both the simple pendulum and the mass/spring system are examples of SHM.

Exercise 8D

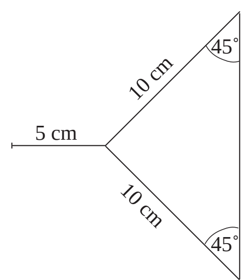
1. A clock manufacturer uses a spring of stiffness 40 Nm^{-1}. It is required that the spring should complete four oscillations every second. What size mass should be attached to the spring? What initial displacement would be required?
2. A 250 gram mass is attached to a spring of natural length 40 cm and stiffness 200 Nm^{-1}. The mass is pulled down 3 cm below its equilibrium position and released. Find
   (a) an expression for its position;
   (b) the period of its motion;
   (c) the amplitude of its motion.
3. A baby bouncer is designed for a baby of average mass 18 kg. The length of the elastic string cannot exceed 80 cm, in order to ensure flexibility of use. Ideally the bouncer should vibrate at a frequency of 0.25 Hz. Determine the stiffness constant of the elastic string.
4. Two identical springs are used to support identical masses. One is pulled down 3 cm from its equilibrium position. The other is pulled down 2 cm from its equilibrium position. How do the amplitude and period of the resulting motion compare?
5. Two identical springs are attached to the 2 kg mass that rests on a smooth surface as shown.



   The springs have stiffness 30 Nm^{-1} and natural lengths 25 cm. The mass is displaced 2 cm to the left and then released. Find the period and amplitude of the resulting motion.
6. A mass/spring system oscillates with period 0.07 s on earth. How would its period compare if it were moved to the moon?
7. The spring in a pinball machine is pulled back with a plunger and then released to fire the balls forward. Assume that the spring and the ball move in a horizontal plane.



   The spring has stiffness 600 Nm^{-1} compressed by 5 cm to fire the ball. The mass of the ball is 50 grams. Find its speed when it leaves contact with the plunger.
8. A catapult is arranged horizontally. It is made from an elastic string of stiffness 80 Nm^{-1}. The diagram below shows the initial dimensions of the catapult, before the elastic is stretched. The stone is placed in the catapult and pulled back 5 cm.



   Find the work done in pulling the catapult back and the speed of the stone when it leaves the catapult. The stone has a mass of 25 grams.

   Find the speed of the stone if the catapult is arranged to fire the stone vertically rather than horizontally.
9. Turbulence causes an aeroplane to experience an up and down motion that is approximately simple harmonic motion. The frequency of the motion is 0.4 Hz and the amplitude of the motion is 1 m. Find the maximum acceleration of the aeroplane.

10. A buoy of mass 20 kg and height 2 m is floating in the sea. The buoy experiences an upward force of $400d$, where d is the depth of the bottom of the buoy below the surface.

(a) Find the equilibrium position of the buoy.

(b) Find the period and amplitude of the motion of the buoy if it is pushed down 0.3 m from its equilibrium position.

11. A metal strip is clamped at one end. Its tip vibrates with frequency 40 Hz and amplitude 8 mm. Find the maximum values of the magnitude of the acceleration and velocity.

8.6 Miscellaneous Exercises

1. A seconds pendulum is such that it takes one second for the pendulum to swing from one end of its path to the other end, i.e. each half of the oscillation takes 1 second.

(a) Find the length of the seconds pendulum.

(b) A seconds pendulum is found to gain one minute per day. Find the necessary change in length of the pendulum if the pendulum is to be made accurate.

2. A simple pendulum oscillates with period $2t$ seconds. By what percentage should the pendulum length be shortened so that it has a period of t seconds?

3. A particle is moving in simple harmonic motion and has a speed of 4 ms^{-1} when it is 1 m from the centre of the oscillation. If the amplitude is 3 m, find the period of the oscillation.

4. A light elastic string with a natural length of 2.5 m and modulus 15 N, is stretched between two points, P and Q, which are 3 m apart on a smooth horizontal table. A particle of mass 3 kg is attached to the mid-point of the string. The particle is pulled 8 cm towards Q, and then released. Show that the particle moves with simple harmonic motion and find the speed of the particle when it is 155 cm from P.

5. A particle describes simple harmonic motion about a point O as centre and the amplitude of the motion is a metres. Given that the period of the motion is $\frac{\pi}{4}$ seconds and that the maximum speed of the particle is 16 ms^{-1}, find

(a) the speed of the particle at a point B, a distance $\frac{1}{2}a$ from O;

(b) the time taken to travel directly from O to B. (AEB)

6. A particle performs simple harmonic motion about a point O on a straight line. The period of motion is 8 s and the maximum distance of the particle from O is 1.2 m. Find its maximum speed and also its speed when it is 0.6 m from O.

Given that the particle is 0.6 m from O after one second of its motion and moving away from O, find how far it has travelled during this one second. (AEB)

7. A particle describes simple harmonic motion about a centre O. When at a distance of 5 cm from O its speed is 24 cm s^{-1} and when at a distance of 12 cm from O its speed is 10 cm s^{-1}. Find the period of the motion and the amplitude of the oscillation.
Determine the time in seconds, to two decimal places, for the particle to travel a distance of 3 cm from O. (AEB)

8. A particle moves along the x-axis and describes simple harmonic motion of period 16 s about the origin O as centre. At time $t = 4$ s, $x = 12$ cm and the particle is moving towards O with speed $\frac{5\pi}{8} \text{ cm s}^{-1}$. Given that the displacement, x, at any time, t, may be written as

$$x = a\cos(\omega t + \phi),$$

find a, ω and ϕ. (AEB)

9. A particle, P, of mass m is attached, at the point C, to two light elastic strings AC and BC. The other ends of the strings are attached to two fixed points, A and B, on a smooth horizontal table, where $AB = 4a$. Both of the strings have the same natural length, a, and the same modulus. When the particle is in its equilibrium position the tension in each string is mg. Show that when the particle performs oscillations along the line AB in which neither string slackens, the motion is simple harmonic with period $\pi\sqrt{\left(\frac{2a}{g}\right)}$.

The breaking tension of each string has magnitude $\frac{3mg}{2}$. Show that when the particle is performing complete simple harmonic oscillations the amplitude of the motion must be less than $\frac{1}{2}a$.

Given that the amplitude of the simple harmonic oscillations is $\frac{1}{4}a$, find the maximum speed of the particle. (AEB)

10. The three points O, B, C lie in that order, on a straight line l on a smooth horizontal plane with $OB = 0.3$ m, $OC = 0.4$ m.

    A particle, P, describes simple harmonic motion with centre O along the line l. At B the speed of the particle is 12 ms^{-1} and at C its speed is 9 ms^{-1}. Find

    (a) the amplitude of the motion;

    (b) the period of the motion;

    (c) the maximum speed of P;

    (d) the time to travel from O to C.

    This simple harmonic motion is caused by a light elastic spring attached to P. The other end of the spring is fixed at a point A on l where A is on the opposite side of O to B and C, and $AO = 2$ m. Given that P has mass 0.2 kg, find the modulus of the spring and the energy stored in it when $AP = 2.4$ m. (AEB)

11. A particle, P, describes simple harmonic motion in the horizontal line ACB, where C is the mid-point of AB. P is at instantaneous rest at the points A and B and has a speed of 5 ms^{-1} when it passes through C. Given that in one second P completes three oscillations from A and back to A, find the distance AB. Also find the distance of P from C when the magnitude of the acceleration of P is $9\pi^2$ ms^{-2}.

    Show that the speed of P when it passes through the point D, which is the mid-point of AC, is $\frac{5\sqrt{3}}{2}$ ms^{-1}. Also find the time taken for P to travel directly from D to A. (AEB)

12. A particle moves with simple harmonic motion along a straight line. At a certain instant it is 9 m away from the centre, O, of its motion and has a speed of 6 ms^{-1} and an acceleration of $\frac{9}{4}$ ms^{-2}.

    Find

    (a) the period of the motion;

    (b) the amplitude of the motion;

    (c) the greatest speed of the particle.

    Given that at time $t = 0$, the particle is 7.5 m from O and is moving towards O, find its displacement from O at any subsequent time, t, and also find the time when it first passes through O. (AEB)