National Certificate of Educational Achievement
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Exemplar for Internal Assessment Resource
Mathematics Level 3

Resource title: Roger's Rabbits

This exemplar supports assessment against:

Achievement Standard 91587

Apply systems of simultaneous equations in solving problems

Student and grade boundary specific exemplar The material has been gathered from student material specific to an A or B assessment resource.

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The task asks students to use the constraints provided to recommend the amount of each type of food that Roger should feed his rabbits to meet their exact daily requirements, and to investigate what quantity of vitamin A would encourage Roger to buy more Zany food.

	Grade Boundary: Low Excellence
1.	For Excellence the student is required to apply systems of simultaneous equations, using extended abstract thinking, in solving problems. This involves devising a strategy to investigate or solve a problem, developing a chain of logical reasoning, forming a generalisation and using correct mathematical statements or communicating mathematical insight. The student has shown evidence of extended and abstract thinking by finding the amount of each type of food to meet the daily requirements **(1)**, and by finding a general solution which satisfies the situation with 6μg of Vitamin A in the Zany product **(2)**. The student has identified an appropriate range of values for the amount Zany for this new situation and has given one possible solution **(3)**. For a more secure Excellence the student would need to accurately communicate their thinking relating to how 6μg of vitamin A in the Zany food relates to the general solution.

x = No. of grams of Xena
y = No. of grams of Yum
z = No. of grams of Zany

The amount of vitamins of each type of food can be represented by the equations

$$2x + 4y + 5z = 1000 \quad (\text{Vit A})$$
$$3x + 7y + 10z = 1600 \quad (\text{vit C})$$
$$5x + 9y + 14z = 2400 \quad (\text{vit E})$$

Solving these gives $x = 300$, $y = 100$, $z = 0$
So to meet the daily requirement Roger should feed them 300 grams of xena, 100 grams of Yum and no Zany 1

If the amount of Vitamin A in Zany changes to 6 micrograms then

$$2x + 4y + 6z = 1000 \quad \text{---- } ①$$
$$3x + 7y + 10z = 1600 \quad \text{---- } ②$$
$$5x + 9y + 14z = 2400 \quad \text{---- } ③$$

Solving these gives no solution.
These equations are inconsistent.

Solving ① ×3 − ② × 2 gives $-2y - 2z = -200$ $\quad y + z = 100$

① ×5 − ③ × 2 gives $2y + 2z = 200$ $\quad y + z = 100$

① ×7 − ②×4 gives $2x \quad + 2z = 600$ $\quad x + \quad z = 300$ 2

So Solution is $300 - z$, $100 - z$, z

if $z > 100$ the amount of Yum would be negative so $0 \leq z \leq 100$ 3

So if $z = 20$ grams, $x = 280$ grams and $y = 80$ grams
ie one solution is 280 grams of Xena, 280 grams of Yum and 20 grams of Zany

If the amount of Vitamin A in Zany is k micrograms

then $2x + 4y + kz = 1000$

$3x + 7y + 10z = 1600$

$5x + 9y + 14z = 2400$

Using the calculator I get 0 micrograms of Zany for lots of values I tried for k except when k = 6 micrograms when there is no unique solution.

Maybe this is because (3) = 4 × (1) − (2) on the previous page.

	Grade Boundary: High Merit
2.	For Merit the student is required to apply systems of simultaneous equations, using relational thinking, in solving problems. This involves selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, and relating findings to a context or communicating thinking using appropriate mathematical statements. The student has shown evidence of relational thinking by finding the amount of each type of food required to meet the daily requirements **(1)**, and indicating that increasing the amount of vitamin A in Zany food does not provide a unique solution **(2)**. The student has identified a possible solution for the amount of each type of food if Zany uses 6μg of vitamin A **(3)**. To be awarded Excellence the student would need to generalise the amount of each type of food required if Zany contains 6μg of vitamin A.

The amount of each vitamin Rogers rabbits need to meet their daily vitamin requirements, and the number of grams of each vitamin in the foods Xena, Yum and Zany can be represented by the following equations where x represents Xena y represents Yum and z represents Zany.

$2x + 4y + 5z = 1000$
$3x + 7y + 10z = 1600$
$5x + 9y + 14z = 2400$
$\rightarrow$ Solved simultaneously
$x = 300$ Xena
$y = 100$ Yum
$z = 0$ Zany.

These calculations lead to the conclusion that in order for his rabbits to meet their exact daily vitamin requirements, Roger should feed them 300 grams of Xena, 100 grams of Yum and 0 grams of Zany each day. Therefore the rabbits daily vitamin requirements can be met by consuming the aforementioned amounts of Xena and Yum alone, Zany is not needed. 1

If Zany increases the amount of vitamin A in their food from 5 micrograms to 6 micrograms, this would change the number of grams of each food Roger should feed his rabbits in order for them to meet their exact daily vitamin requirem-ents.

$2x + 4y + 6z = 1000$ (1)
$3x + 7y + 10z = 1600$ (2)
$5x + 9y + 14z = 2400$ (3)

(1) × 1.5 $\quad 3x + 6y + 9z = 1500$ (4)
$\quad 3x + 7y + 10z = 1600$ (2)
(2) − (4) $\quad y + z = 100$

(1) × 2.5 $\quad 5x + 10y + 15z = 2500$ (5)
$\quad 5x + 9y + 14z = 2400$ (3)
(3) − (5) $\quad -y - z = -100$

There are many solutions to the number of grams of each food Roger should now feed his rabbits in order to meet their daily vitamin requirements. There is no one real solution. 2

No amount of vitamin A would encourage Roger to buy more Zany food because his rabbits daily vitamin intake is already met by Xena and Yum, ~~and~~ so he does not need any Zany.

One example of a possible solution of the number of grams of each food roger should now feed his rabbits is 3

x = 250 grams Xena
y = 50 grams Yum
z = 50 grams Zany.

	Grade Boundary: Low Merit
3.	For Merit the student is required to apply systems of simultaneous equations, using relational thinking, in solving problems. This involves selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, and relating findings to a context or communicating thinking using appropriate mathematical statements. The student has shown evidence of relational thinking by finding the amount of each type of food required to meet the daily requirements **(1)**, and by identifying that the change to 6μg of vitamin in Zany food produces no unique solutions **(2)**. For a more secure Merit the student would need to provide a possible solution which meets the new situation and accurately communicate what was being calculated at each step.

Student 3: Low Merit

Vitamin A $2x + 4y + 5z = 1000$

Vitamin C $3x + 7y + 10z = 1600$

Vitamin E $5x + 9y + 14z = 2400$

$x = 300$

$y = 100$

$z = 0$

If Roger wants his rabbits daily vitamin intake to be 1000μg of vitamin A,

1600 g of vitamin C and 2400 g of vitamin E, In order to meet these exact daily vitamin requirements Roger should feed his rabbits 300 grams of Xena feed and 100 grams of Yum feed. 1

Vitamin A $2x + 4y + 6z = 1000$

Vitamin C $3x + 7y + 10z = 1600$ 2

Vitamin E $5x + 9y + 14z = 2400$

These equations are inconsistant and there is no exact solutions.

	Grade Boundary: High Achieved
4.	For Achieved the student is required to apply systems of simultaneous equations in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. The student has shown evidence of applying systems of simultaneous equations by forming the equations **(1)** and using them to find a solution, and making an appropriate recommendation regarding the amount of each type of food required **(2)**. To be awarded Merit the student would need to consider how the amount of each type of food would change if the number of micrograms of vitamin A in the Zany food changes to 6.

Student 4: High Achieved

xena	yum	zany	

$$2A + 4C + 5E = 1000$$
$$3A + 7C + 10E = 1600$$
$$5A + 9C + 14E = 2400$$

1

$x = 300g$
$y = 100g$
$z = 0g$

Dear rodger

I recomend that you feed your rabbits 300g of xena, 100g of yum and 0g of zany rabbit food, to reach their exact daily vitamin request.

2

	Grade Boundary: Low Achieved
5.	For Achieved the student is required to apply systems of simultaneous equations in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. The student has shown evidence of applying simultaneous equations methods by providing the equations for each vitamin **(1)**, and by solving them to find a solution **(2)**. For a more secure Achieved the student would need to indicate more accurately what is represented by each variable and interpret the solution in context.

Student 5: Low Achieved

Vitaman E = e
Vitaman C = c
Vitaman A = a

Xena contains
$2a + 3c + 5e$

Yum
$4a + 7c + 9e$

Zany
$5a + 10c + 14e$

A $2x + 4y + 5z = 1000$

C $3x + 7y + 10z = 1600$

E $5x + 9y + 14z = 2400$

1

$x = 300$
$y = 100$
$z = 0$

2

	Grade Boundary: High Not Achieved
6.	For Achieved the student is required to apply systems of simultaneous equations in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. The student has provided the equations for each of the vitamins **(1)**. To be awarded Achieved the student would need to correctly solve these equations.

Student 6: High Not Achieved

Xena 2µg A
3mg C
5mg E

Yum 4µg A
7mg C
9mg E

Zany 5µg A
10mg C
14mg E

Roger wants

$$\boxed{\begin{array}{ll} 1000\,\mu g & A \\ 1600\,mg & C \\ 2400\,mg & E \end{array}}$$

Equation.

		xena	yum	zany	
1	A	2	4	5	= 1000
2	C	3	7	10	= 1600
3	E	5	9	14	= 2400

Equation

1. $2x + 4y + 5z = 1000$
2. $3x + 7y + 10z = 16000$
3. $5x + 9y + 14z = 2400$

1