National Certificate of Educational Achievement
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Exemplar for Internal Assessment Resource
Mathematics Level 3

Resource title: Elaine's Equations

This exemplar supports assessment against:

Achievement Standard 91587

Apply systems of simultaneous equations in solving problems

Student and grade boundary specific exemplar The material has been gathered from student material specific to an A or B assessment resource.

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The task asks students to form systems of equations by using three different methods to create the third equation. Students are also asked to solve these systems of equations giving a geometric interpretation for each and writing a general statement about each solution.

	Grade Boundary: Low Excellence
1.	For Excellence the student is required to apply systems of simultaneous equations, using extended abstract thinking, in solving problems. This involves devising a strategy to investigate or solve a problem, developing a chain of logical reasoning, forming a generalisation, and using correct mathematical statements or communicating mathematical insight. The student has solved and geometrically interpreted the solutions for all three methods (1). There is evidence of extended and abstract thinking in the student's general comment on the expected solution when generating a third equation using unconnected coefficients (2). For a more secure Excellence the student would need to make a general statement about the solutions for the other two systems of equations.

Student 1: Low Excellence

$2x+2z=3y+1 \quad = 2x-3y+2z=1$

$y-4z=8$

• Method One - Pin Number

$3x+2y+8z=7$

$2x-3y+2z=1$

$y-4z=8$

so $6x+4y+16z=14$

$-\ 6x+9y+6z=3$

$13y+10z=11$

now $13y-52z=104$

$-\ 13y+10z=11$

$-62z=93$

$z=-1.5$

so $y-4\times-1.5=8$

$y=2$

and $2x-3\times2+2\times-1.5=1$

$2x-9=1$

$x=5$

So $x=5$
$y=2$
$z=-1.5$

1

This means there is a unique solutions. The three ~~points~~ planes intersect at one point giving a unique value for x, y and z as there is only one point of intersection of the 3 planes.

These equations describe planes in 3D as they have 3 variables. When I solved the equations I was looking for the intersections of these planes. In this method the new equation was formed by using random numbers that didn't seem related to the other equations and in this case the solution was a point. I think this will always be the ~~s~~case when the third equation is made this way. 2

• Method 2 - Multiply by 3

$2x-3y+2z=1$) $\times 3$

$y-4z=8$

$6x-9y+6z=3$

now $6x-9y+6z=3$

$-\ 6x-9y+6z=3$

$0=0$ 1

So two planes have the same equation, this means they are the same on the graph. The third plane intersects along a line with these 2 planes giving infinite solutions. The system is dependant.

• Method 3 - Changing constant

$2x-3y+2z=1$

$y-4z=8$

$2x-3y+2z=6$

So $2x-3y+2z=6$

$-\ 2x-3y+2z=1$

1 $0=5$

So two planes are parallel ∴ these two planes do not intersect at all. The third plane intersects these two planes but as there is no point where all 3 planes intersect, there are no solutions and the system is inconsistent.

	Grade Boundary: High Merit
2.	For Merit the student is required to apply systems of simultaneous equations, using relational thinking, in solving problems. This involves selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, and relating findings to a context or communicating thinking using appropriate mathematical statements. There is evidence of relational thinking by the student solving and geometrically interpreting the solutions for methods 1 and 3 (1). The student has correctly recognised the dependent equations in method 2, but has incorrectly interpreted these geometrically as a triangular shape (2). To reach Excellence the student would need to solve and interpret the system of equations for all methods and make a general statement about the solution set for each system of equations.

Student 2: High Merit

① $2x + 2z = 3y + 1$ — original

$y = 4z + 8$

$2x - 3y + 2z = 1$ $\times 3$

$y - 4z = 8$ $\times 13$

~~$2x + z$~~ $3x + 2y + 8z = 7$ $\times 2$

$6x - 9y + 6z = 3$

$- \; 6x + 4y + 16z = 14$

$-13y - 10z = -11$

$13y - 52z = 104$

$+ \; -13 \; - 10z = -11$

$-62z = 93$

$z = -1.5$

$y - 4(-1.5) = 8$

$y - -6 = 8$

$y = 2$

$2x - 3(2) + 2(-1.5) = 1$

$2x - 6 - 3 = 1$

$2x = 10$

$x = 5$

1 $(5, 2, -1.5)$ — Unique solution — Consistant

②

$2x - 3y + 2z = 1$ $\times 3$

$y - 4z = 8$

$6x - 9y + 6z = 3$

$6x - 9y + 6z = 3$

$- \; 6x - 9y + 6z = 3$

$0 = 0$

1

— 3 lines of solutions

— Dependant

③ — 2 parallel planes

$2x - 3y + 2z = 1$

$y - 4z = 8$

$2x - 3y + 2z = 6$

$2x - 3y + 2z = 1$ $\neq$

$- \; 2x - 3y + 2z = 6$

$0 = -5$

1

~~— 2 lines of solutions~~

— Inconsistant

For the first set of equations, it is possible to solve, resulting in a unique solution that is consistant. The planes ~~[illegible]~~ will intersect at points $(5, 2, -1.5.)$

For the second set of equations, attempting to solve will result in two $6x - 9y + 6z = 3$ equations $(0 = 0)$. This will resemble ~~two~~ planes always being cut by two planes making a triangular shape. This is dependant.

2

For the third set of equations, attempting to solve will result in two equations with same coefficients but different constants $(0 = -5)$. This is inconsistant and will resemble two parallel planes both being cut by a third plane.

	Grade Boundary: Low Merit
3.	For Merit the student is required to apply systems of simultaneous equations, using relational thinking, in solving problems. This involves selecting and carrying out a logical sequence of steps, connecting different concepts or representations, demonstrating understanding of concepts, and relating findings to a context or communicating thinking using appropriate mathematical statements. There is evidence of relational thinking by the student solving and geometrically interpreting the solutions for methods 2 and 3 (1). The student has not completely solved the system of equations for method 1 (2). For a more secure Merit the student would need to solve the system of equations for all methods.

1. $2x + 2z = 3y + 1$ | $2x - 3y + 2z = 1$
2. $y = 4z + 8$ | $y - 4z = 8$

3a. $3x + 2y + 8z = 7$
3b. $6x + 6z = 9y + 3$ | $6x - 9y + 6z = 3$
3c. $2x + 2z = 3y + 6$ | $2x - 3y + 2z = 6$

a.
1. $2x - 3y + 2z = 1$ ×2
2. $y - 4z = 8$ ×2
3. $3x + 2y + 8z = 7$
1×2. $4x - 6y + 4z = 2$
2×2. $2y - 8z = 16$
1+2. $4x - 5y = 10$ ×4
2+3. $3x + 4y = 23$ ×5
$16x - 20y = 40$
$15x + 20y = 115$
$31x = 155$
$x = 5$
$y = -2$
$z =$

1. $2x - 3y + 2z = 1$ ×2 ×4
2. $y - 4z = 8$ ×2
3. $3x + 2y + 8z = 7$
4. $4x - 6y + 4z = 2$
5. $2y - 8z = 16$
6. $8x - 12y + 8z = 4$
4+2. $4x - 5y = 10$
5+3. $3x + 4y = 23$
6+3. $5x - 14y = -3$
$x = 5$
$y = -2$
incosistant z

2

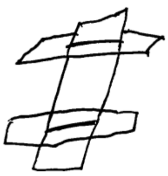
b.
1. $2x - 3y + 2z = 1$ ×3
2. $y - 4z = 8$
3. $6x + 6z - 9y = 3$
1×3. $6x - 9y + 6z = 3$
$0 = 0$

1

Two planes are the same, 1 and 3. which means that there are infinite solutions along the line of intersection with plane 2. The planes are dependant.

c.
1. $2x - 3y + 2z = 1$
2. $y - 4z = 8$
3. $2x - 3y + 2z = 6$
1−3 $0 = -5$

1



2 planes are parallel to each other 1 and 3. This causes the equations to be inconsistant so there is no solution.

	Grade Boundary: High Achieved
4.	For Achieved the student is required to apply systems of simultaneous equations in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. There is evidence of applying systems of simultaneous equations by the student solving the equations for methods 1 and 2 (1) and interpreting the solutions for method 1 geometrically (2). To reach Merit the student would need to solve and provide the geometric interpretation for both methods 1 and 2.

Student 4: High Achieved

Method 1

Equation 1 $\quad 2x + 2z = 3y + 1 \quad = \quad 2x + 2z - 3y = 1$

Equation 2 $\quad y = 4z + 8 \quad = \quad y - 4z = 8$

Equation 3 $\quad 3x + 2y + 8z = 7$

$2x + 2z - 3y = 1 \quad \times 3$

$6x + 6z - 9y = 3$

$x = 5$

$y = 2$

$z = -1.5$ 1

$3x + 2y + 8z = 7 \quad \times 2$

$6x + 4y + 16z = 14$

$6x + 4y + 16z = 14$

$- \cancel{6x + 6z - 9}$

$6x - 9y + 6z = 3$

$13y + 10z = 11$

$y - 4z = 8 \quad \times 13$

$13y - 52z = 104$

$- \; 13y + 10z = 11$

$-62z = 93$

$z = -1.5$

$y - 4z = 8$

$y - 4(-1.5) = 8$

$y + 6 = 8$

$y = 2$

$2x + 2z = 3y + 1$

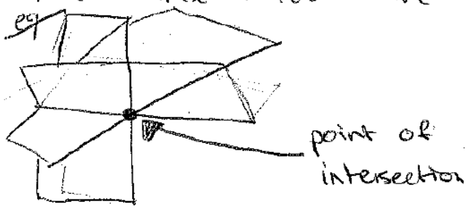
$2x + 2(-1.5) = 3(2) + 1$

$2x - 3 = 6 + 1$

$2x = 10$

$x = 5$

In method 1 as there is ~~an~~ one set answer for x, y, z graphicly the three lines have one point where they all cross. eg



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method 2

Equation 1 $\quad 2x + 2z = 3y + 1 \qquad = 2x + 2z - 3y = 1$

Equation 2 $\quad y = 4z + 8$

Equation 3 $\quad 6x + 6z = 9y + 3 \qquad = 6x + 6z - 9y = 3$

$$2x + 2z = 3y + 1 \quad \times 3$$
$$6x + 6z = 9y + 3$$

$$- \quad 6x + 6z - 9y = 3$$
$$6x + 6z - 9y = 3$$

$$0 = 0$$

1

Infininte solutions

	Grade Boundary: Low Achieved
5.	For Achieved the student is required to apply systems of simultaneous equations in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. There is evidence of applying simultaneous equations methods by the student solving and geometrically interpreting the system of equations for method 1. For a more secure Achieved the student would need to solve or provide a geometric interpretation for another system of equations.

Student 5: Low Achieved

Original Equations	Rearranged 2 given equations
1) $2x + 2z = 3y + 1$	1) $2x - 3y + 2z = 1$
2) $y = 4z + 8$	2) $y - 4z = 8$

Method 1: $3x + 2y + 8z = 7$

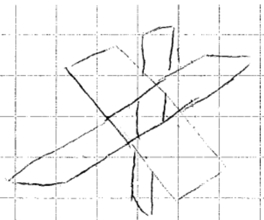
$$2x - 3y + 2z = 1$$
$$y - 4z = 8$$
$$3x + 2y + 8z = 7$$

$x = 5$

$y = 2$

$z = -1.5$

Method one!



One unique solution is given at the point where all 3 equations intersect.

	Grade Boundary: High Not Achieved
6.	For Achieved the student is required to apply systems of simultaneous equations in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. The student has solved the system of equations for method 1. To reach Achieved the student would need to provide a geometrical interpretation of this solution.

Student 6: High Not Achieved

$2x + 2z = 3y + 1$

$y = 4z + 8$

$3x + 2y + 8z = 7$

$$\begin{cases} 2x - 3y + 2z - 1 = 0 & -(1) \\ y - 4z - 8 = 0 & -(2) \\ 3x + 2y + 8z - 7 = 0 & -(3) \end{cases}$$

$(1) \times 6$, $12x - 18y + 12z - 6 = 0$ $-(4)$

$(3) \times 4$, $12x + 8y + 32z - 28 = 0$ $-(5)$

$(4) - (5)$, $-26y - 20z + 22 = 0$ $-(6)$

$(2) \times 5$, $5y - 20z - 40 = 0$ $-(7)$

$(6) - (7)$, $-31y + 62 = 0$

$y = 2$

sub $y = 2$ to (2)

$(2) - 4z - 8 = 0$

$(2) - 4z = 8$

$-4z = 6$

$z = -1.5$

sub $y = 2$, $z = -1.5$ to (5)

$12x + 8(2) + 32(-1.5) - 28 = 0$

$12x + 16 - 48 - 28 = 0$

$12x - 60 = 0$

$12x = 60$

$x = 5$

$\therefore y = 2 \quad z = -1.5 \quad x = 5$