National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

# Exemplar for Internal Assessment Resource

# Mathematics Level 3

## Resource title: Polar ice

This exemplar supports assessment against:

Achievement Standard 91580

Investigate time series data

| Expected responses |
| --- |
| The moderators have developed expected student responses from a wide variety of sources |

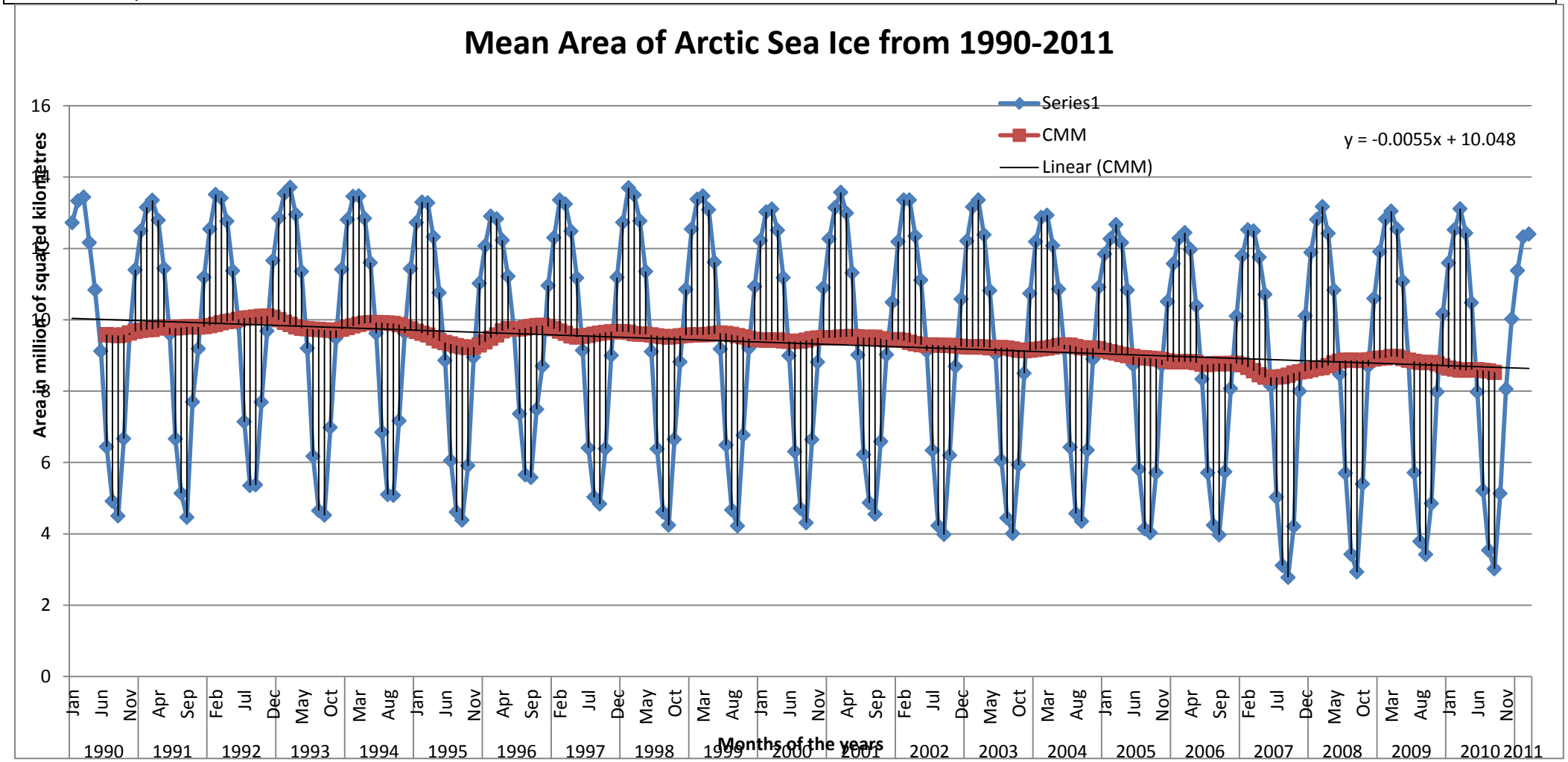
Date version published by Ministry of Education: December 2012
To support internal assessment from 2013

| | Grade Boundary: Low Excellence |
|---|---|
| 1. | For Excellence the student is required to investigate time series data with statistical insight. This involves integrating statistical and contextual knowledge throughout the statistical enquiry cycle and reflecting on the process.<br><br>There is evidence that the student has researched the context and used this to develop a clear purpose for the investigation (1).<br><br>The appropriateness of the model used to make forecasts has been discussed based on the reliability trend and the seasonal effects (2).<br><br>How the forecasts might be used and who could potentially use the forecasts has been identified and discussed (3).<br><br>There is evidence of statistical and contextual knowledge being integrated throughout the script with the evaluation of the adequacy of the fitted model (4).<br><br>There is consideration of other variables and the comparison with the Antarctica time series (5).<br><br>For a more secure Excellence, the student would need to investigate the data more fully, for example they could link the percentage increase and decrease figures found to the data to see if they are similar or investigate other variables such as depth of ice or temperature.<br><br>The robustness of the model used for forecasting could be tested by omitting the last three consecutive values, refitting the model, and comparing the predictions with the actual values. |

Student 1: Low Excellence

It has been speculated in the media that the area of sea ice on the planet is decreasing. Is this actually happening? If so what is causing such changes? How fast is the decrease? What would this mean for the area of sea ice in the Arctic? Would the habitats of animals such as polar bears be destroyed? The natural resources defence council (NRDC) report (http://www.nrdc.org/globalwarming/qthinice.asp) states that although local temperatures fluctuate naturally, over the past 50 years the average global temperature has increased at the fastest rate in recorded history. Other impacts will include sea level rise, decreases in the amount of sea ice and more frequent and intense storm events.
I am going to investigate whether or not there is a decreasing trend in the area of sea ice in the Arctic and if so what is the rate of such a change. I will then make a comparison with the Antarctica data.

1




This graph shows the mean area of Arctic Sea Ice, measured in million $km^2$, from January 1990-March 2011. The data (mean area of Arctic sea ice) is recorded monthly. The mean area of the Arctic Sea Ice is calculated from satellite images.

**Features of the data**

The Arctic Sea Ice is the result of the sea water in the Arctic Ocean freezing when its temperature drops below $-2^0$C (freezing point of sea water).
The mean area of Arctic Sea Ice (per month) fluctuated between approximately 2.3 million squared kilometres to 14 million squared kilometres over this time.
There is a seasonal pattern apparent in the data, with a period of one year. The mean area of Arctic Sea ice highest during the months February-March and lowest during the months of August-September. This seasonal pattern in the data corresponds to changes in seasons throughout the course of the year. The summer months in the northern hemisphere (Jun-Aug) would generally yield warmer temperatures, leading to the sea ice in the Arctic Ocean melting, resulting in lower mean area of Arctic Sea Ice. And the winter months in the Arctic region (Dec-Feb) would generally have lower temperatures, hence more sea water could freeze and form sea ice, this could be the reason for a higher area of Arctic Sea Ice during that part of the year.
There is an overall gradual decreasing trend of the mean area of Arctic Sea Ice in the last 21 years (approximately) that data has been recorded. This is shown on the graph with the trend line having a negative gradient. The gradient of the trendline suggest that the mean area of the Arctic Sea Ice has decreased at a rate of 5500 $km^2$ each month between Jan 1990-Mar 2011, on average.
The variability between the highest annual mean area of Arctic Sea Ice (peak) and the lowest annual mean area of the Arctic Sea Ice (trough)appears to be higher towards the more recent years (e.g 2010,2009 and 2008) than in the earlier years. (1990,1991,1992).

**Fit of the model**

The fitted linear model to the CMM appears to be a relatively good fit as the trendline for most of the time period is very close to the CMM. The exceptions would be as mentioned previously, the second half of 1995 and 2007 where the trendline is above the CMM line. I tried to research possible reasons for these observations and found a report from New Scientist online that identified and commented a record low in the area of Arctic Sea ice in 2007 also. 2

**Forecast:**

The linear model (trend) appears to fit the Centred Moving Mean (CMM) quite closely. Therefore I think the linear model that I am using is reasonably reliable to use to make forecasts. If this model is extrapolated then the eventual result will be no sea ice in the Arctic at all, however we are assuming that this will not be the case for the time periods we are forecasting for.

y =Trend value Seasonal effect for June
June 2011 (x=258)
y =(-0.0055(258)+10.048)+(-0.30646)
=8.32254

I predict that an estimate of the mean area of Arctic Sea would be 8.3 Million $km^2$ in June 2011, assuming the same pattern continues. Looking at the individual seasonal effects for June, there is little variation from year to year as they range from -0.0054 to -0.62. Therefore the average seasonal effect used in the calculation of my forecast for June should be reasonably reliable which means the forecast for June 2011 should be as well. 2

Dec 2012 (x=276)
Y= trend value + Seasonal effects for Dec
=(-0.0055(276)+10.048)+(1.502)
=10.032
I predict that an estimate of the mean area of Arctic Sea Ice would be 10.03 million square kilometres in December 2012, assuming the same pattern continues. There is relatively little variation between the individual seasonal effects from year to year for December as they only range from 1.06 to 1.73. Hence the seasonal effect for December should be quite reliable when used to calculate the forecasts.

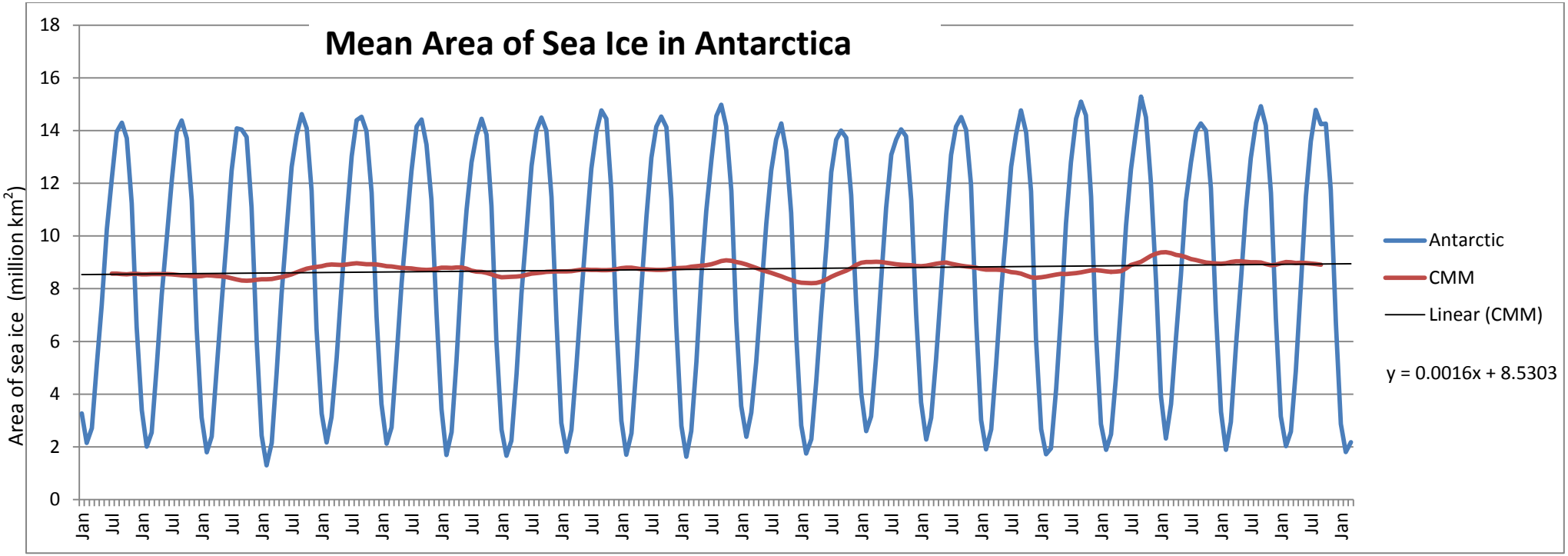
This forecast would be of interest to scientists studying the climatic processes of the Earth, Environmental Groups (e.g Greenpeace) as well as scientist studying animal which lives at the Arctic Region. As the area of Arctic sea ice decreasing would have an impact on the climate of the region, possibly contributing to global warming as the melting of sea ice mean that there is more heat absorbed by the sea water than previously, heating up the ocean. The Arctic Sea Ice is the habitat of Polar Bears and would have an impact on how the polar bears live. Environmental groups may want to alert the general public about the decline in the Arctic Sea Ice to get people to change their lifestyles to lessen their carbon footprints.

3

The more recent trend in the centred moving mean appears to have a more rapidly decreasing trend than the trend line. (the centred moving mean line from late 2008 to 2011 seem to form a line that has a steeper negative gradient than those of the trend line). This could mean that the forecasts that I am making are more optimistic. I could have used separate straight lines to model the trend for each of the periods of increasing trend and decreasing trends throughout the 21 years so that the trend line more closely approximate the trend shown in the centred moving mean.

4

My findings of a gradual decrease in the mean area of sea ice in the Arctic are partially supported by other research from NASA which states the area is decreasing by about 4% per decade. They also identified a record low for the mean area of sea ice in the Arctic in 2007 but a possible reason for this was not identified.




We do not know if there are any other factors apart from rising global temperatures that have an impact on the melting of the Arctic sea ice. Looking at the data collected from on the mean area of the Antarctic Sea Ice, the area is relatively constant during the period January 1990 to March 2011 as the trend has remained about 8.5 million $km^2$. When comparing the trend for each of these data sets, the seasonal pattern looks almost symmetrical opposite. So maybe there could be other factors that are causing the mean area of the Arctic Sea Ice to decrease, just as there could be other factors to explain the mean area of sea ice being relatively constant in Antarctic despite also being impacted by the rise in global temperatures.

A NASA report ( http://www.nasa.gov/topics/earth/features/antarctic_melting.html) mentions that the area of sea ice in Antarctic sea ice is increasing by approximately 1% per decade and Arctic sea ice is decreasing approximately at 4% per decade. Flooding sea ice from rising sea levels and ozone depletions were possible as reasons for the different trends. The combination of these factors has resulted in colder, stormier weather and faster winds that create open water which in turn promotes production of sea ice which could explain the 1% increase for the Antarctic but 1% is very small over a decade so could be with the margin of error anyhow.

While the general decreasing trend in the mean area of the Arctic Sea ice seem to suggest that the global temperatures are rising, causing the Arctic Sea Ice to melt at a faster rate. However, only the mean area was recorded in both sets of data, it would perhaps be better if the depth of the sea ice was also taken into consideration, so that the volume of the sea ice is analysed. (as we might not be able to assume that the sea ices' depth is changing at the same rate as the area is decreasing.)

| | Grade Boundary: High Merit |
|---|---|
| 2. | For Merit the student is required to investigate time series data with justification. This involves linking components of the statistical enquiry cycle to the context and making supporting statements which refer to evidence.<br><br>There is evidence that the student has researched the context and used this to develop a clear purpose for the investigation (1).<br><br>Comments on how accurate the predictions and prediction intervals are have been based on the goodness of fit of the model and on an inspection of the residual plot (2).<br><br>Comments about there being an odd residual for September 2007 need to be developed (3).<br><br>To reach Excellence more evidence of integrating statistical and contextual knowledge is needed, for example the student could investigate the Antarctic sea ice or global temperatures to see if the pattern in the Arctic sea ice is similar.<br><br>Comments about the reliability of the seasonal effects used when making forecasts need to be supported with evidence. |

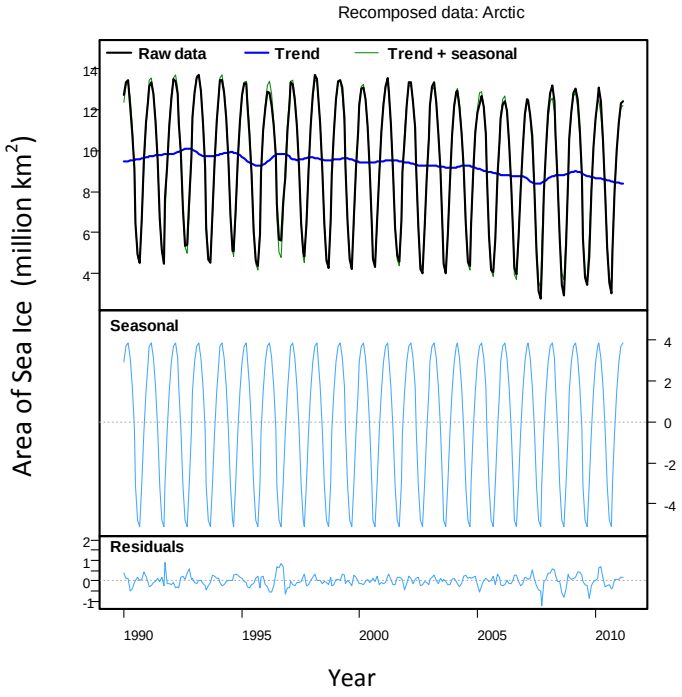
Student 2: High Merit

The two data sets given are about the sea ice located in the polar ice caps (Arctic and Antarctica). Polar ice caps simply mean a high altitude region in a planet or natural satellite that is covered in ice. There is no specific size nor composition for the mass of ice to be given the term "polar ice cap" for this case, the data that I've chosen is about the polar ice caps in Arctic which is mainly composed of sea ice. Sea ice is just ice made from sea, but in the process it loses its composites and eventually becomes fresh water. Since sea water is denser that fresh water so the freezing point for sea water is below 0 degrees Celsius.

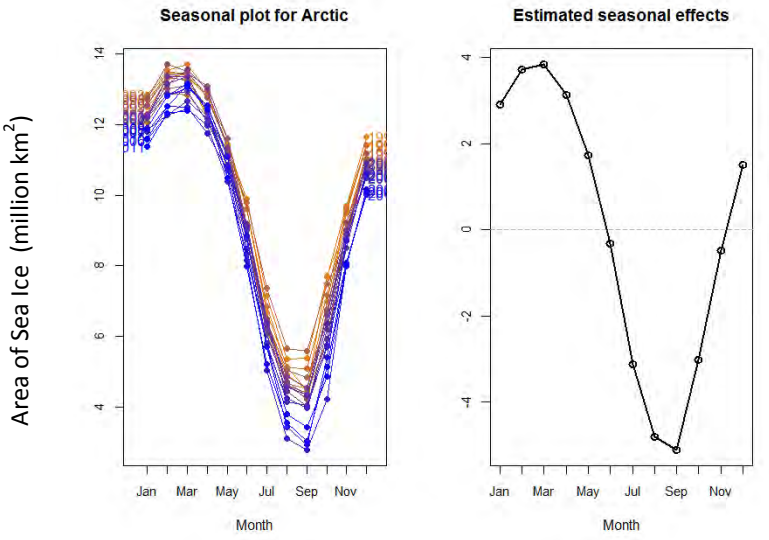
It is widely known that the phenomenon of global warming is happening on our planet. This is means that the average temperatures of the Earth's atmosphere and oceans are increasing. The effects of an increase in global temperature mean that sea levels rise and the amount of sea ice decreases as a result. According to Wikipedia, warming is expected to be strongest in the Arctic and would be associated with the continuing retreat of glaciers and sea ice. It would be interesting to investigate whether the area of sea ice is in fact decreasing in the Arctic, at what rate, any other interesting patterns or features and who or what will be affected by such decreases.

1

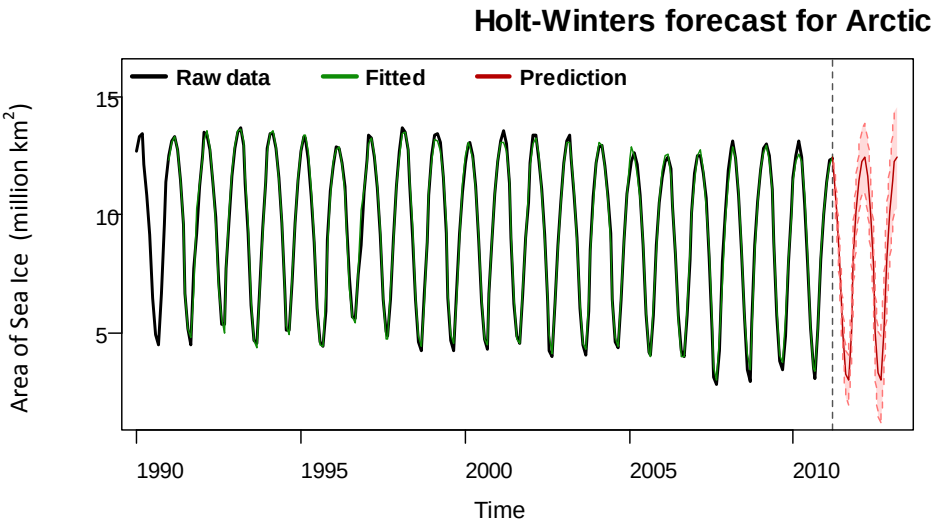
**Mean Area Sea Ice (million square kilometres) for Arctic Graphs**




The graph shows data about the mean area of sea ice in the Arctic. Looking at the raw data the mean area of sea ice in Arctic fluctuated between 13.5 (million square kilometres) and 2.5 (million square kilometres) between 1990 to 2011. The graph of the smoothed trend shows a very gradual decreasing trend. It appears that the mean area of sea ice has decreased on average from about 9.5 million square kilometres to about 8.5 million square kilometres from January 1990 to March 2011.




The graph of the estimated average seasonal effects shows a seasonal pattern with the maximum mean area of sea ice in the Arctic occurring at March which peaks at around 3.8 million square kilometres above the trend line and the minimum mean area of sea ice in the Arctic occurring at September which is at the lowest at about 5million square kilometres below the trend line. This pattern corresponds to the seasons experienced at the Arctic which is in the Northern Hemisphere. The temperatures during March, which near the end of Winter start of Spring, in the Arctic are relatively lower causing more ice to form and the temperatures during September, which is near the end of Summer start of Autumn, are relatively high causing more ice to melt.




The following predictions and prediction intervals were produced from iNZight.

| Month | Lower limit | Prediction | Upper limit |
|---|---|---|---|
| January 2012 | 10.1 | 11.5 | 12.8 |
| February 2012 | 10.8 | 12.3 | 14 |
| March 2012 | 10.9 | 12.4 | 13.9 |

If this trend continues, my prediction for the mean area of sea ice in arctic in March 2012 is 12.4 million $km^2$.and is estimated to be between 10.9 and 13.9 million $km^2$.

The model fitted appears to be reasonably good as the differences between the raw and fitted data in the graph above are appear to be very small and my prediction is not far from the given data. There is little variability in the seasonal effects from year to year and the seasonal pattern has stayed fairly constant over the whole time so I can rely on the predictions generated from the Holt-Winters model.
Looking at the graph of the residuals also confirms that the predictions should be fairly reliable as the residuals appear to be small as most are no more than 0.5 million $km^2$.above or below the line and have limited variability and no obvious pattern. 2

The residual at September 2007 is a bit odd compared to the others as it is nearly 1 million $km^2$.below the line which is quite a bit more compared to the others. 3

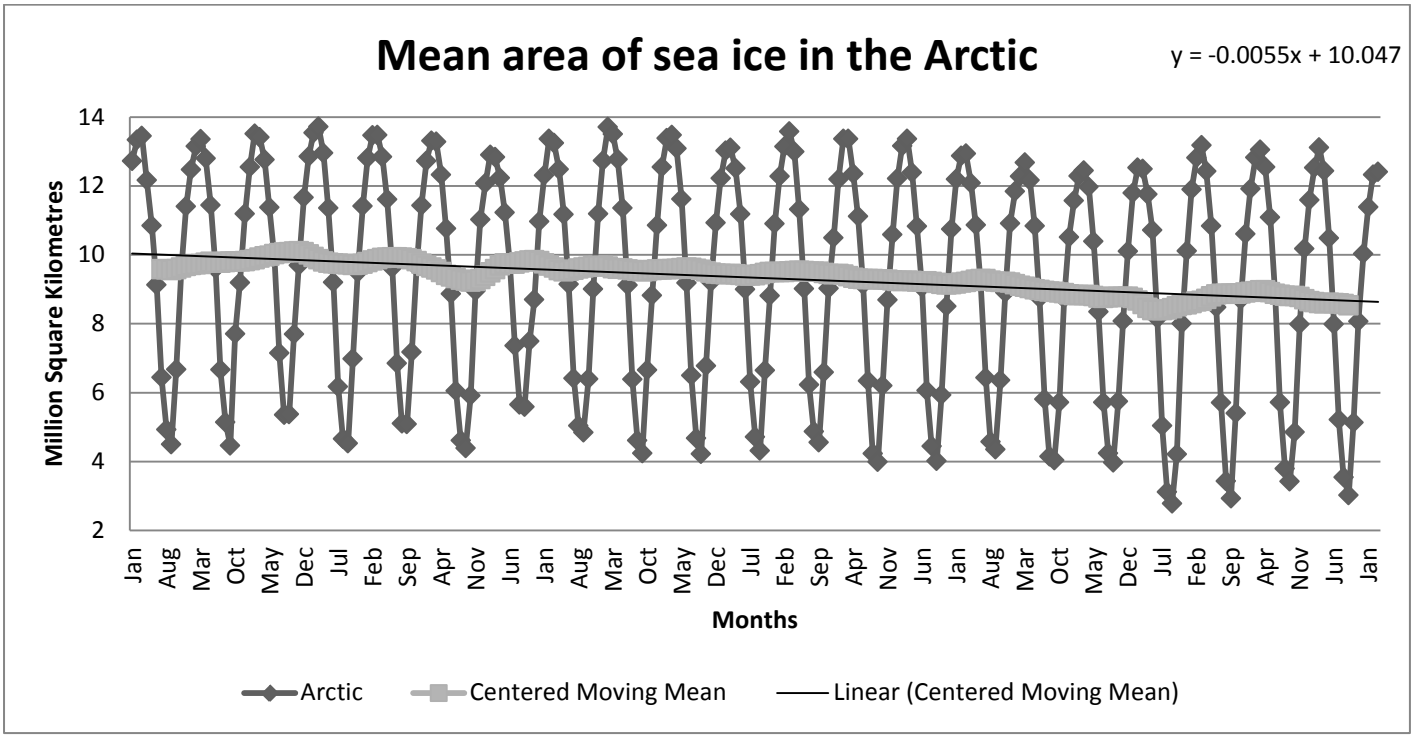
However this graph does not take account of that fact that the global temperature is also increasing due to global warming therefore this could impact area of sea ice in the Arctic. The decreasing area of sea ice in the arctic could be a result from global warming where the temperature is increasing globally causing more ice to melt in warmer seasons and less ice forming in the colder seasons in the Arctic. The decreasing area of sea ice in the Arctic could impact the global weather. The reduction of could sea ice could result in more area of less reflective sea water beneath exposed. This causes less solar heat to be reflected back into space and could possibly increase the effects of global warming and could also possibly increase the trend shown in the graph. Scientist such as geologist would be interested in this data since it can give them predictions of the area of sea ice so they can relate them to the global weather change in the past 20 years. Biologists would also like to know the rate of decrease in the area of sea ice in the Arctic as this could mean the wildlife such as polar bears will no longer have a place to live. They may be able to predict form the trend approximately when this may be the case.

| | Grade Boundary: Low Merit |
|---|---|
| 3. | For Merit the student is required to investigate time series data with justification. This involves linking components of the statistical enquiry cycle to the context and making supporting statements which refer to evidence.<br><br>There is evidence of investigating time series data with justification as components of the statistical enquiry cycle have been linked to the context.<br><br>There is evidence of some research related to the purpose of the investigation (1).<br><br>Features of the data have been identified with descriptions of the trend and seasonal pattern which have been related to the context (2).<br><br>The student has used a correct model to make forecasts which have been given in context and appropriately rounded (3).<br><br>There is some discussion on the accuracy of the forecasts (4).<br><br>For a more secure Merit comments about the reliability of the forecasts need to be further developed. |

It is well known that global warming is occurring on our planet. Global warming is the rise in the average temperature of the Earth's atmosphere and oceans. These temperature rises could then cause devastating effects to the wildlife and their habitats on Earth. There have been reports that the area of sea ice in the Arctic has been decreasing because of global warming and so could mean that the habitats of the polar bears that live in the Arctic could be diminished fairly soon. I am going to investigate the mean area of sea ice in the Arctic to see if this decrease over time is actually happening and how fast it is occurring. 1

**Features of the raw data**

The raw data shows the mean area of sea ice in the Arctic is slightly decreasing due to global warming. Polar sea ice helps reflect solar heat back into space, making the air colder, while reducing heat loss from the water below the ice and controlling currents. However $CO^2$ emissions create a carbonated cloud which blocks the sun rays from exiting the earth causing global warming. The mean area of the Arctic was calculated from daily satellite images from January 1990 to March 2011.




There is a Seasonal pattern happening to the mean area of sea ice in the Arctic as the peaks occur during the Winter months (Jan-Apr) when the temperatures are lower whereas the troughs occur during the Summer months (Jul-Oct) when the temperatures are higher. This is because sea ice freezes during the cold winter months and it shrinks during the summer months as the sea ice melts.

There is a pretty stable variability from the mean area of sea ice data in the arctic as it appears to only be slightly decreasing in value over time.

The gradient of the trendline suggests that the mean area of sea ice in the arctic is slightly decreasing at a rate of 66,000 square kilometres per year on average.

There are no known spikes shown in the graph.

2

**Forecasts**

Sept 2011, x = 261

Y = -0.0055 x 261 + 10.047 -5.10 = 3.5115

My prediction for September 2011 suggests that the estimate for the mean area of sea ice in the Arctic is 3.51 million square kilometres.

March 2012, x = 267

Y = -0.0055 x 267 + 10.047 + 3.84 = 12.4185

My prediction for Mar-12 suggests that the estimate for the mean area of sea ice in the Arctic is 12.42 million square kilometres. 3

Since the trend line appears to fit the smoothed data reasonably well with very little deviation away from the CMM line (apart from April-July 1995 and April-July 2007) so my predictions should be fairly are reliable. 4

The data from satellite observations of the mean area of sea ice in the Artic is covered for 20 years of monthly data and used to make predictions for the further years and therefore making my predictions more accurate.

The data for mean area of sea ice in the Arctic only shows the surface mean area of the Arctic. However this is not enough to tell if global warming is affecting it as you may need to know the depth of the Arctic sea ice as well as the temperature of the water below the ice. Also there should be data for $CO^2$ emissions as it controls how harsh the summer months are getting when the sun rays gets trapped inside the earth, and therefore accelerating the process when the polar ice caps melt.

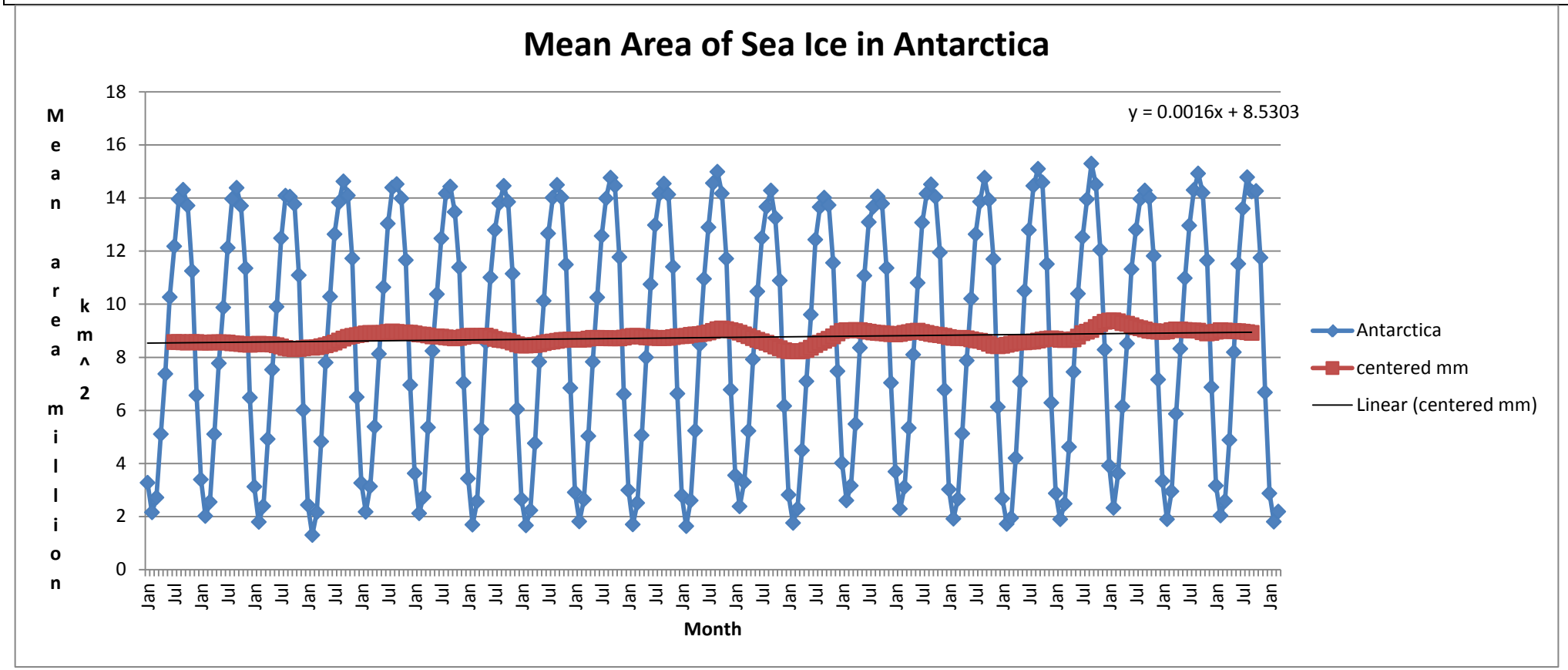
| | Grade Boundary: High Achieved |
|---|---|
| 4. | For Achieved the student is required show evidence of using each component of the statistical enquiry cycle to investigate time series data.<br><br>There is evidence of some research related to the purpose of the investigation (1).<br><br>Features of the data have been identified with descriptions of the trend and seasonal pattern. The description of the seasonal pattern has not been clearly related to the context (2).<br>The model has been used to make correct forecasts, but they have not been appropriately rounded (3).<br><br>To reach Merit appropriately rounded forecasts and a comment on the accuracy of the forecasts is needed. The comment "*I expect my predictions to be reasonable"* is insufficient and would need to be developed to support the accuracy of forecasts.<br><br>The description of the seasonal pattern would also need to be more clearly related to the context with an understanding of the seasons. |

Student 4: High Achieved

Sea ice forms from salt water, in the polar regions of the Earth that has cooled to its freezing temperature. The salt water that forms sea ice in the southern ocean has a high concentration of salt which results in it having a freezing point of -1.8deg Celsius. In the Antarctic sea ice is extremely seasonal covering up to 15.5 million square km in winter and decreasing to about 1.2 million square km in summer. In Antarctica the thicker, floating ice shelves are gradually melting to be replaced by far thinner sea ice cover, this means that the overall thickness of the sea ice is decreasing.

I am going to investigate what is happening to the area of sea ice in Antarctica to see if it is increasing, decreasing or no change at all.

1




The mean area of sea ice fluctuates between 1.3 million square km and 15.29 million square km between the time periods of Jan 1990 through to march 2011. On average the mean area of sea ice increases very slightly over the recorded time. The centred moving mean was calculated and graphed. By visual reference the CMM does not show any significant change. However the CMM gives an almost horizontal trend.

2

A trend line was added with an equation of Y= 0.0016X+8.5303 this shows that there is a very slight increase in the mean area of sea ice in Antarctica. The equation suggests that the increase is 1600 square km on average each month.

This data shows a strong seasonal pattern with peaks in the mean area of sea ice in Antarctica in September with an exception in August 2010 and the troughs in February. There is a small dip occurring from May 2001 to December 2002.

2

Forecasts :-

September 2011:- 0.0016(260)+8.5303 = 3.165327776

3.16532776+5.780972222 = 14.72727222

February 2012:- 0.0016(265)+8.5303 = 8.9543

8.9543-6.8115625 = 2.1427375

I predict that the amount of Arctic sea ice for September 2011 will be 14.72727222 million square km.

3

The amount of Arctic sea ice for February 2012 will be 2.1427375 million square km. Provided the current trend prevails. I expect my predictions to be reasonable as I have 20 years and three months of monthly data to base my predictions on.

My model can be improved by adding two different trend lines instead on one. If we add a separate trend line to the data from 2001 to 2011 i judge that the trend could have a lower gradient than the previous data (1990 to 2001). This would be due to the slight dip in the centred moving mean between 2001 and 2002. If I use the 2001 trend line to make my predictions I could have predictions lower than they currently are. These predictions may be more reliable as they have been based on more recent data, which should be given more weight than older data.

These forecasts would be of use to agencies such as the ministry of environment, or NASA as well as environmental groups such as Green Peace.

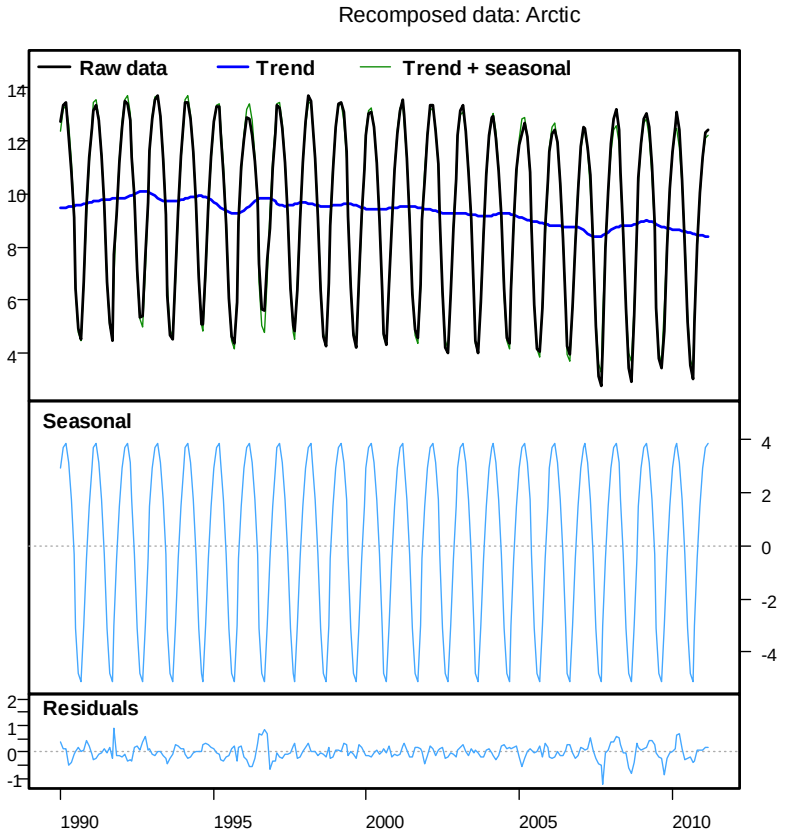
The data that I was given takes no account of the thickness of the sea ice. Therefore if someone was trying to investigate whether Antarctica was losing or gaining sea ice volume they would not be able to make an accurate judgement also the rate of melting of the sea ice cannot be calculated. The data also does not take into account the density of sea ice covering the surface of the ocean i.e. is all 14 million square km covered in one slab of ice or is only 90% covered in sea ice, this would mean that even though the data is showing that the mean area of sea ice is increasing actual volume of sea ice could be decreasing. The data also does not state the average temperature during each month. Therefore we cannot compare the change in temperature compared to the changes in sea ice area. The concentration of salt water and freezing temperature of the southern ocean is unknown. As a result there will be an unknown effect on the area of sea ice which we cannot calculate due to the lack of data. These factors may result in my forecasts being less valid.

| | Grade Boundary: Low Achieved |
|---|---|
| 5. | For Achieved the student is required show evidence of using each component of the statistical enquiry cycle to investigate time series data.<br><br>There is evidence of investigating time series data in the use of each component of the statistical enquiry cycle.<br><br>The student has selected a variable to investigate and used iNZight software to select and use appropriate displays and find an appropriate model (1).<br><br>Features of the data have been identified with descriptions of the trend and seasonal pattern, but these have not been clearly related to the context (2).<br><br>The Holt-Winters model has been used to make a forecast (3).<br><br>For a more secure Achieved there needs to be a better overall understanding of the context. In the description of the trend the student needs to refer to the decrease in the mean area of sea ice in the Arctic as an 'average' effect. Graphs should have axes labelled with units and it is also expected there would be evidence of research into the context. |

Student 5: Low Achieved

Sea ice is present in the Arctic region of the globe, and is formed from the freezing of sea water. Since the oceans consist of sea water, sea ice freezes at a temperature of -1.8C. The amount of sea ice in the Arctic is apparently decreasing over a period of 21 years due to causes such as global warming and the Greenhouse effect. I am going to see if this decrease is actually happening and if so how fast.

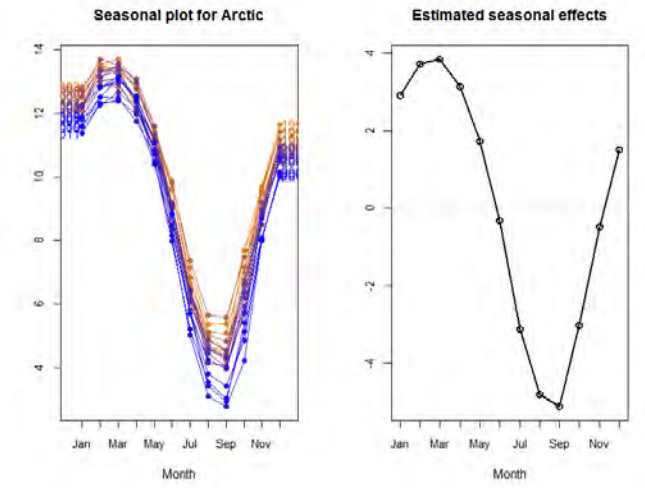
Arctic Sea Ice Graphs



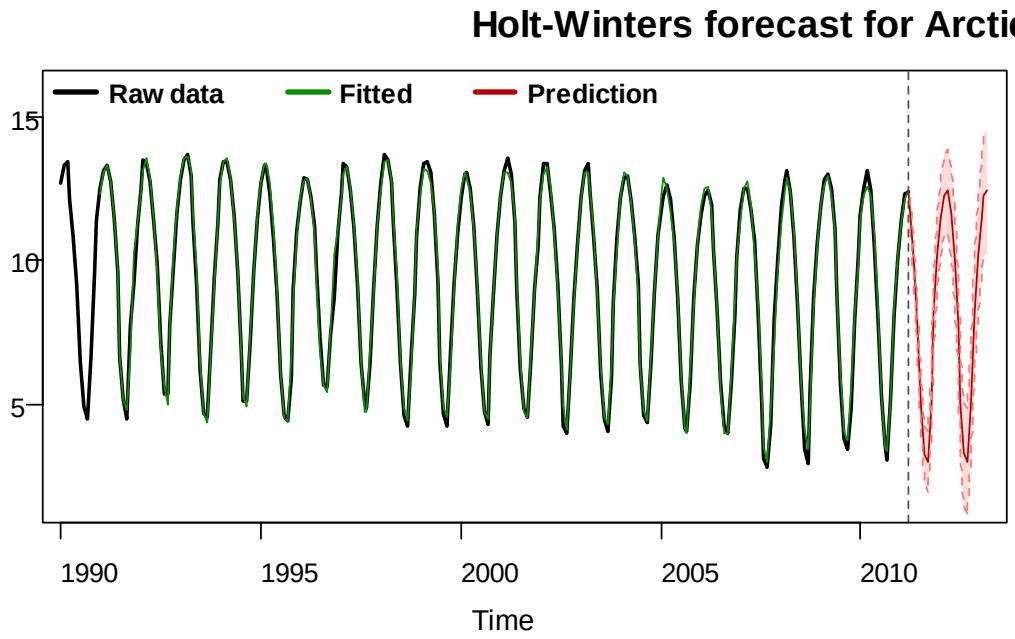

1

This graph shows the amount of sea ice (in millions of square kilometres) in the Arctic regions from January 1990 to March 2011. The amount of sea ice in the Arctic over a period of 21 years is generally decreasing. The area of sea ice has fallen from about 9.5 million of square kilometres to about 8.5 million of square kilometres over this time.

2



There is an annual seasonal pattern that can be seen in the graph. Generally, the month with the highest amount of sea ice in the Arctic is March and the month with the lowest amount of sea ice in the Arctic is September. 2




We are able to make quite accurate predictions about the amount of sea ice in the future because of the fit of the trend line to the data and using the average seasonal effects for each month. I estimate from the graph that the area of Sea Ice for Sept 2011 will be about 3 million of square kilometres. 3

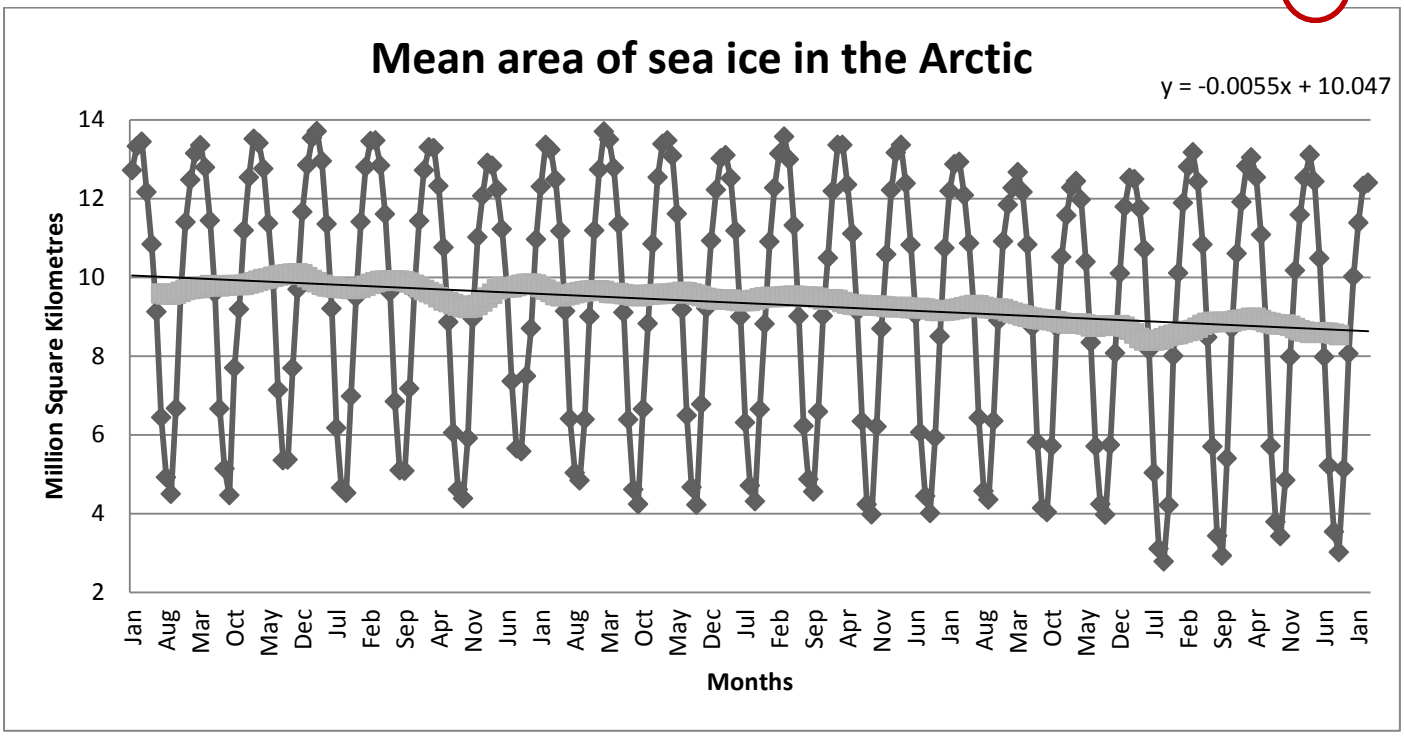
The projection of the amount of sea ice in Sept 2011 should quite accurate, assuming the trend line does not change and there are no errors or ramps in the data set.

| | Grade Boundary: High Not Achieved |
|---|---|
| 6. | For Achieved the student is required show evidence of using each component of the statistical enquiry cycle to investigate time series data.<br><br>The student has selected a variable to investigate, selected and used an appropriate display and found an appropriate model (1).<br><br>The interpretation of the gradient of the trend equation is incorrect (2).<br><br>Seasonal peaks in the data have been identified, but the seasonal pattern hasn't been clearly described (3).<br><br>The model has been used to make a consistent forecast (4).<br><br>There is no purpose for the investigation or evidence of using the research into the context. To reach Achieved the student would need to accurately describe the trend and relate this to the context. |

The issue of Gobal warming is a problem that has been happening for over the past 20 years now. It is caused by the temperatures rising over the earth over the period of time. This mainly happens in the arctic and in antartica since the temperatures are much warmer there. If the Ice caps melt this will cause the water levels to rise which will affect the land area and also a few places maybe under water due to this. So Global warming is a huge environmental issue.

This time series graph shows the area of sea ice in millions of square kilometres in the Arctic between January 1990 to March 2011 (approximately over the last 20 years).

1




The equation of the trendline is y = -0.0055x + 10.047 which suggests that the gradient of the trend line of the arctic sea ice is decreasing at a rate of 0.55 per year. 2

The time series graph shows a very seasonal cycle and there isn't any ramps or random shown on the graph, which means that the monthly area of sea ice in Arctic is very consistent and not random.

Each season has the same pattern, the earlier months in the year Feb and March are the peaks in the graph and seasonal effects which have the highest area of sea ice (millions of square kilometre) and Sept in the graph have the lowest area of sea ice in millions of square kilometres 3

Using the model to forecast the area of sea ice in the Arctic for May 2011

Trend Value = y = -0.0055x + 10.047

=-0.0055*(257)+ 10.047

=8.6335

seasonal effect for the month of May: 1.747542

Forecast = 8.6335 + 1.747542

= 10.38104167

If this trend carries on the prediction that the area of sea ice in millions of square kilometres for May 2011 will be 10.38 4