## Exemplar for Internal Assessment Resource Mathematics Level 3

## Resource title: Food for Thought

This exemplar supports assessment against:
Achievement Standard 91580
Investigate time series data

## Expected responses

The moderators have developed expected student responses from a wide variety of sources

[^0]|  | Grade Boundary: Low Excellence |
| :--- | :--- |
| 1. | For Excellence the student is required to investigate time series data, with statistical insight. <br> This involves integrating statistical and contextual knowledge throughout the statistical enquiry <br> cycle and reflecting on the process. |
| There is evidence that the student has researched the context and used this to develop a clear <br> purpose for the investigation (1). |  |
| The appropriateness of the model used to make predictions has been discussed, with an <br> evaluation of the adequacy of the model and a discussion of the reliability of the calculated <br> seasonal effects (2). <br> There is evidence of statistical and contextual knowledge being integrated. There is a <br> consideration of other variables and the comparison, with the population time series and money <br> spent on takeaway food time series (3). <br> Actual and predicted values as well as prediction intervals have been provided for three <br> consecutive quarters. A comparison of these has been made to confirm the robustness of the <br> fitted model (4). <br> For a more secure Excellence comments about why the residuals for Q1 2004 and Q1 2008 are <br> unusual would need to be developed, as well an interpretation of the prediction intervals. |  |

It has been widely publicised that both NZ and the rest of the world have recently experienced a global financial crisis towards the late 2000's. This has meant that most of society then had to closely consider how much money they were unnecessarily spending, where could they cut back and what was essential or not.

According to a report from the Ministry for the Environment dated the end of April 2009 which is on the website http://www.mfe.govt.nz/environmental-reporting/consumption/household-expenditure/by-category/, the top three household consumption expenditure categories were food and beverages, housing, and transport. It also states that New Zealand's total household consumption expenditure continued to increase but the increases were not as large as in previous years.

I am going to investigate the amount of money spent in supermarket and grocery stores from 2000 to 2010 to see whether or not the overall trend is actually increasing and if there is anything unusual happening in the data. It would be interesting compare this time series with expenditure in other retail sectors.


Looking at the smoothed decomposed data, there is an obvious increase in the amount of money spent at NZ supermarket and grocery stores from 2000 to the end of 2010. There is a steady increase from 2000 to towards the end of 2007 but then there is a quite a sharp fall in the trend from here till the end of 2009. The trend seems to plateau at the end of 2010 to about $\$ 2500$ million per quarter. The trendline suggests that total money spent in NZ supermarket and grocery stores has increased on average from approx. $\$ 1900$ million dollars per quarter to $\$ 2500$ million per quarter over the 2000 to 2010 period.

There may have been some sort of down turn in the economy e.g global financial crisis, that could have accounted for such a sharp fall towards the end of 2007. I would need to research this further to see if these two events coincided and compare to see if the same decrease in spending occurred on takeaway food at the same time.

The seasonal plots show that there is an obvious seasonal pattern in the money spent on food in NZ supermarket and grocery stores with the highest sales being in Q4 each year. The average seasonal effects show that Q4 is the quarter when the most money is spent as it is about $\$ 170$ million above the trend line. This of course makes sense since this quarter corresponds to the months October to December where people begin to start their Christmas shopping in preparation for Christmas dinner and fill up Christmas stockings with extra goodies. This end of this quarter is also the start of the holiday period so more money is probably spent at supermarkets and grocery stores because of social functions that occur during this time more so than the other quarters. Quarters2 and 3 which goes from April to June for Q2 then Jul-Sept for Q3 are the time when the least amount of money is spent as they are both about $\$ 80$ million below the trend line.


A possible reason for the increase in amount of money spent on food in NZ supermarket and grocery stores is that the population in NZ also increased during this time as shown by the population graph of NZ above. Since there were more people in NZ there were more people to go into such stores and so there was more money to spend.

The appearance of the population graph and supermarket and grocery stores graph are similar at the beginning as they are both quite level. But the population graph continues to increase and doesn't level off towards the end as the supermarket and grocery store graph does.

Holt-Winters forecast for Superma


Using the holt-winters model for calculating forecasts I estimate that the total amount of money spent on food in Supermarket and Grocery in NZ in Q4 2010 is 2770 million dollars but could be between $\$ 2666$ and $\$ 2867$ million dollar, for Q4 2011 the total amount spent is predicted to be $\$ 2810$ million dollars but could be between $\$ 2614$ and $\$ 3008$ million dollars.

Using such a model to make forecasts assumes that the seasonal pattern of total amount of money spent in Supermarket and Grocery in NZ is reasonable constant and not too varied. The fitted model fits the data fairly well (apart from just after 2004 and 2008) and the seasonal effects have remained relatively constant but with some small variation in the first and last quarters. Therefore I am reasonably confident that my forecasts are accurate.

Looking at the residual plot most residuals are within $\$ 50$ million of the trendline. The overall range of the raw data is $\$ 884$ million (maximum - minimum $=2756-1872$ ) so a residual of $\$ 50$ million is quite small in proportion to $\$ 884$ million. The exception would be the unusual residuals from Q1 2004 which is right on the $\$ 50$ million mark and Q1 2008 which is about $\$ 75$ million above the trendline. There is also a bit of a difference between the raw and fitted data for these times looking at the holt-winters plot.

Below is a comparison of the actual data values and predicted values for the last three consecutive quarters of the recorded data. To calculate the predictions, the model was refitted and the predictions re-calculated with the last three data points removed.

| Date | Actual data | Prediction | Lower Limit | Upper Limit |
| :--- | :--- | :--- | :--- | :--- |
| Q1 2010, | $\$ 2526$ | $\$ 2595$ | $\$ 2493$ | $\$ 2697$ |
| Q2 2010 | $\$ 2468$ | $\$ 2488$. | $\$ 2362$ | $\$ 2615$ |
| Q3 2010 | $\$ 2471$ | $\$ 2505$ | $\$ 2356$ | $\$ 2654$ |

All actual data values (which are in millions of dollars, rounded to the nearest dollar) are in the prediction intervals which means that the fitted model is a good fit and we can rely on the forecasts produced.


Both graphs show a slight growth and then they both start to increase quite sharply, with spending at supermarket and grocery stores from increasing 2001 and spending on takeaways from 2002. A decrease in spending occurred in supermarket and grocery stores from the end of 2007 start of 2008 but spending on takeaways decreased from 2007. The seasonal pattern for each quarter is much more varied. These are similar timeframes so perhaps there was a reason for this. Finance companies in NZ started to collapse and go into receivership around these times e.g. Bridgecorp, Capital and Merchant with huge losses. There was also a global financial crisis that occurred at this time which meant that people would not have as much money to spend on themselves especially luxuries such as takeaways. This could explain the trend we see in the data.

|  | Grade Boundary: High Merit |
| :--- | :--- |
| 2. | For Merit the student is required to investigate time series data with justification. This involves <br> linking components of the statistical enquiry cycle to the context and making supporting <br> statements which refer to evidence. <br> There is evidence that the student has researched the context and used this to develop a clear <br> purpose for the investigation (1). <br> A comment on how accurate the forecast is and why it may be an overestimate has been made <br> based on how well the trend line fits the smoothed data (2). <br> While consideration has been given to an alternative model, there is no explanation as to why <br> such a model is required for data from 2007. Any comment would need to include a link to <br> recalculating the forecast (3). <br> To reach Excellence the communication of findings would need to include more contextual <br> knowledge about spending on food in supermarket and grocery stores in NZ and the money <br> spent on fresh meat, fish, poultry and fruit and vegetables. |

The data we have been provided with is from the Retail Trade Survey (RTS) which was a survey undertaken by Statistics New Zealand. I have chosen to investigate the spending in supermarket and grocery stores in NZ data set to see if there are any noticeable trend(s) and features in this data set. I will then compare this investigation with the amount on money spent on fresh meat, fish, poultry and fruit and vegetables to see if they have similar trends I have read a report from the Statistics New Zealand website
(http://www.stats.govt.nz/browse for stats/people and communities/Households/HouseholdEconomicSurvey M RYeJun10.aspx) that stated that from June 2007 till June 2010 the average weekly household expenditure increased as well as the average annual household income from regular source. Therefore I would expect the amount of money spent in supermarket and grocery stores in NZ to have increased over time.

Supermarket and Grocery Stores Spending in NZ


The smoothed data (centred moving mean) shows an increase in money spent in supermarket and grocery stores in New Zealand. The trendline shows that the amount spent is increasing by an average of $\$ 20$ million per quarter over the whole series.

There is a seasonal pattern in the data with amount of money spent in NZ at supermarket and grocery stores being the highest in the fourth quarter (Oct-Dec) which makes sense since this contains the Christmas period and lows in quarters two (Apr-Jun) and three (Jul-Sep). Quarter one (Jan-Mar) has occasional highs and occasional lows. The third and second quarter lows are of similar values whereas when quarter one is high it still has a significantly lower value than quarter four. The first quarter generally has a low seasonal effect. There are no spikes or ramps in this series.

An explanation for the increased spending at supermarket and grocery stores during the fourth quarter is easily explained. This is the Christmas and holiday period where people are getting prepared for Christmas dinners, barbeques, parties, stocking fillers etc so naturally are going to spend more on food and drink.
Our method of analysis gives equal weighting to all of the data values whether they be the more recent or older data. Forecasting using a model that gives more weighting to the more recent data maybe more appropriate use of more recent seasonal variations may be more appropriate than giving equal weighting to all seasons. However since the fitted trend line does not appear to fit the CMM that well from 2007 this is most likely not appropriate.

A prediction can be made using: Trend (20.024x + 1856.3) + Average Seasonal Effect
I estimate a forecast for the amount of money spent on food in supermarket and grocery stores in New Zealand for the year 2011 in the fourth quarter.

2011 Q4 is the $48^{\text {th }}$ quarter in the series; the average seasonal effect for the fourth quarter is 173.71 .
$20.024 \times 48+1856.3+173.71=2991.162$. ie. Approximately $\$ \mathbf{2 9 9 0}$ million will be spent on food in supermarket and grocery stores in New Zealand.

The average seasonal effect for the $4^{\text {th }}$ quarter which is used in the calculation of the forecast above is calculated by taking an average of all the $4^{\text {th }}$ quarter seasonal effects. These range from $\$ 148.83$ million to $\$ 206.78$ million which is a relatively small range and the individual seasonal effects for the $4^{\text {th }}$ quarters are relatively stable fr $\mathrm{m}_{2}$ year to year. Therefore the estimate of the average seasonal effect for the $4{ }^{\text {th }}$ quarter used to calculate in the forecast calculation should be reliable.
The trendline does not fit the smoothed data (centred moving mean) well visually from the end of 2006 onwards. The trendline shows a continuous increase despite the fact that the CMM is starting to level off from 2008 onwards and is quite a bit below the fitted trendline at the end. So my prediction of the amount of money spent in NZ at supermarket and grocery stores may be an overestimate.

To more accurately predict future amounts of money spent on food supermarket and grocery stores in New Zealand it would be appropriate to have a separate model from late 2007 onwards to use for future predictions.



In comparison, the money spent on fresh meat, fish, poultry and fruit and vegetables follows a very similar trend. The increase in spending is much more gradual as the trendline shows that the amount spent is increasing by an average of $\$ 1.1$ million per quarter over the whole series the money spent on fresh meat, fish, poultry and fruit and vegetables whereas it is $\$ 20$ million for supermarket and grocery stores in. Places that sell fresh food possible only target only certain groups of society whereas almost everyone would use a supermarket which could explain the difference in spending.

|  | Grade Boundary: Low Merit |
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| 3. | For Merit the student is required to investigate time series data with justification. This involves <br> linking components of the statistical enquiry cycle to the context and making supporting <br> statements which refer to evidence. |
| There is evidence of investigating time series data with justification in the use of each <br> component of the statistical enquiry cycle. <br> There is evidence of some research related to the purpose of the investigation (1). <br> Features of the data have been identified with descriptions of the trend and seasonal pattern <br> and the student has accurately related these to the context (2). |  |
| The student has identified the change in the smoothed data, but has not related it to the <br> forecast (3). |  |
| The student has used a correct model make a forecast which has been given in context and <br> appropriately rounded (4). <br> A comment on the accuracy of the forecast has been given (5). |  |
| For a more secure Merit the comment on the accuracy of the forecast needs to be developed. <br> The student could consider whether or not the CMM dropping below the fitted trend line towards <br> the end of the graph would potentially have an effect on the estimated value of the forecast. |  |

## Time Series Assessment - Supermarket Spending

There has been a lot of talk in the media in recent times that even though we have been through tough economic times in NZ we still continue to spend money at an increasing rate. Therefore I am going to investigate whether the amount of money spent in NZ supermarket and grocery stores is in fact on the increase over time. Even when money is tight we still need to buy food etc. so I would expect that the pattern in the data will be increasing as has been broadcasted. However from researching on the internet, $N Z$ did recently experience a global financial crisis (GFC)so it will be interesting to see if this affected the amount of spending.


Overall the amount of money spent in NZ at supermarket and grocery stores has increased over the whole 11 years. The trendline suggests that the average increase in the amount of money spent per quarter is $\$ 20.024$ million over the whole period.

The centred moving mean shows sales increasing from the third quarter 2000 up to quarter four 2007. At this point the CMM decreases until quarter four 2008 where it levels off.

There is an obvious seasonal pattern with the largest amount of money spent in NZ on tood in supermarket and grocery stores in the fourth quarter which are the months from October-December and lows in the second quarter each year which are the months April-June. The fourth quarter being the highest can be explained by the fact that this quarter corresponds to the Christmas and holiday period when spending on food and drinks always high. People at this time of the year tend to have more social outings e.g end of year work parties, people over for BBQs since it is the holidays etc so therefore the need for food and drink at these sort of events increases.

An estimate for the amount of money spent in NZ on food in supermarket and grocery stores in the third quarter 2012 is:
$y=20.024 x+1856.3+$ ASE
The third quarter is the $51^{\text {st }}$ quarter.
$=20.024 \times(51)+1856.3-83.07$
$=\$ 2794$ million


Generally the linear model is a reasonably good fit to the CMM data. On the graph, the centred moving mean remains close to the trendline until near the end of 2007. Although at the end of the graph, the trendline continues to increase whereas the centred moving mean is levelling out at this point on the graph so therefore I cannot be too sure whether my prediction for the third quarter 2012 is entirely accurate.

|  | Grade Boundary: High Achieved |
| :--- | :--- |
| 4. | For Achieved the student is required to show evidence of using each component of the <br> statistical enquiry cycle to investigate time series data. <br> There is evidence of some research related to the purpose of the investigation (1). <br> The student has selected a variable to investigate, and has used iNZight software to select and <br> use appropriate displays and find an appropriate model (2). <br> Features of the data have been identified with descriptions of the trend and seasonal pattern, <br> and the student has accurately related these to the context. A decreasing ramp in the data <br> between 2008 and 2009 has been identified as a feature in the data (3). |
| The model has been used to make a correct forecast. The link of the forecast to the context <br> needs to refer to the amount of money spent in supermarket and grocery stores, rather than <br> how many sales there will be (4). <br> To reach Merit the student needs to make a comment on the accuracy of the forecasts. |  |

Maths Time Series Internal
My investigation is into NZ grocery and supermarket sales from 2000 to 2010. I am going to see if there are any trends over time and especially if there are any particular seasons when New Zealanders spend more or less at the supermarket. Research I have read from newspapers and from watching TV suggests that there seems to be more spending each year so my investigation may or may not verify this.

Recomposed data: Supermarket.and.grocery.stores


These graphs show that the average amount of money spent in NZ supermarket and grocery stores has an increasing trend with a slight decrease between 2008 and 2009. The trend starts from about $\$ 1950$ million in 2000 with the trend then levelling off at around $\$ 2500$ million per quarter in 2010. As the data gets to 2008, we see a slight decreasing ramp in the graph. This enables us to see that during this time something may have occurred and is why the amount of sales is slighting decreasing, or it may have just been a bad year.

The average estimated seasonal effects graph shows the seasonal pattern in the graph. Quarter four, Oct-Dec, has the highest peak as it is about $\$ 170$ million above the trend line showing that it is a particularly good part of the year with Christmas shopping etc while the other quarters 2 (AprJun)and quarter 3 (Jul-Sep) are the lowest points as they are about $\$ 80$ million below the trend line. It is not surprising that quarter 4 , which contains the Christmas shopping period, would be the quarter where the most money is spent. This is because people seem to spend more money on food and alcohol for events such as Christmas dinners, parties, end of year work shouts and of course heading into the New Year period.

Holt-Winters forecast for Superma


Looking at the above graph which seems fairly accurate, the prediction for how many sales there will be in NZ supermarkets and grocery stores in quarter four of 2011 is between $\$ 2600$ and $\$ 3000$ million dollars.

|  | Grade Boundary: Low Achieved |
| :--- | :--- |
| 5. | For Achieved the student is required to show evidence of using each component of the <br> statistical enquiry cycle to investigate time series data. <br> There is a purpose for the investigation, but no evidence of any research into the context (1). <br> The student has selected a variable to investigate, selected and used an appropriate display <br> and found an appropriate model (2). <br> The trend and seasonal pattern have been identified, but not clearly related to the context (3). <br> The model has been used to make a correct forecast (4). |
| There is evidence of investigating time series data in the use of each component of the <br> statistical enquiry cycle. <br> For a more secure Achieved, features of the data need to be more clearly related to the context. |  |

I am going to investigate the amount of money spent in supermarket and grocery stores in NZ to see what the trend in the data is and at what times of the year people spend the most.

## Supermarket and grocery

 stores MM 1,902.80 1,871.90 $1,935.00$1,877.80 1,935.83

2,087.50 1,941.53 1,938.68 1,906.10 $\quad 1,946.23 \quad 1,943.88$ $1,894.70 \quad 1,947.30 \quad 1,946.76$ 1,896.60 1,958.00 1,952.65 2,091.80 $1,969.68 \quad 1,963.84$ $\begin{array}{lll}1,948.90 & 1,990.55 & 1,980.11\end{array}$ 1,941.40 $\quad 2,015.85 \quad 2,003.20$ 1,980.10 $\quad 2,042.28 \quad 2,029.06$ $\begin{array}{lll}2,193.00 & 2,063.03 & 2,052.65\end{array}$ 2,054.60 $\quad 2,076.33 \quad 2,069.68$ $\begin{array}{llll}2,024.40 & 2,129.38 & 2,102.85\end{array}$ $\begin{array}{lll}2,033.30 & 2,184.15 & 2,156.76\end{array}$ $\begin{array}{lll}2,405.20 & 2,217.33 & 2,200.74\end{array}$ 204.46

Q4

Q1
Q2
Q3
Q4
Q1
Q2
Q3
Q4
Q1
Q2
Q3
Q4
Q1
Q2
Q3


1

1
2


1







Graph:


There is an increasing trend in the graph. Over the 10 years the sales in NZ supermarket and grocery stores is increasing by an average of $\$ 20$ million per quarter. There is also a seasonal pattern occurring, with highs in sales every quarter 4 and lows in sales around every quarter 2. Quarter 3 is also quite close to being the lowest sales period.

The trend line fits the data reasonably well as it lies along the data points fairly well in the beginning, but is inaccurate towards the end where the points level off.

Using the equation $y=20.024 x+1856.3+$ average seasonal effect (-5.92), my prediction in quarter 1 , $2013(x=53)$ will be $\$ 2911.652$ millions of dollars.

|  | Grade Boundary: High Not Achieved |
| :--- | :--- |
| 6. | For Achieved the student is required to show evidence of using each component of the <br> statistical enquiry cycle to investigate time series data. <br> The student has selected a variable to investigate, selected and used an appropriate display <br> and found an appropriate model (1). <br> The student has identified a feature in the data by describing the trend in context (2). <br> The quarter number in the forecast calculation is incorrect (3). <br> There is no purpose for the investigation or evidence of research into the context. To reach <br> Achieved the evidence for identifying features in the data would need to include identifying the <br> seasonal pattern. The model needs to be used to make a correct forecast. |

I am looking at NZ's Supermarket and Grocery store spending from 2000 to 2010.


## Supermarket and Grocery Stores



The trendline is a good fit to the data. We can see that the money spent in NZ supermarket and grocery stores is increasing each quarter by $\$ 21.59$ million.

Forecast for quarter 1 in 2011.
$Y=21.588 x+1832.8-3.48$
$Y=21.588 \times 46+1832.8-3.48$3
$Y=\$ 2822.4$ million
I am reasonably confident with my prediction as the trend line follows the data very closely. This would make it easy and reliable to forecast future quarters as the prediction is highly likely to be very similar to what will actually happen.


[^0]:    Date version published by
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