National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Exemplar for Internal Assessment Resource

Mathematics Level 3

Resource title: Maths End Ferris Wheels

This exemplar supports assessment against:

Achievement Standard 91575

Apply trigonometric methods in solving problems

Expected responses

The moderators have developed expected student responses from a wide variety of sources

---

Date version published by
Ministry of Education

December 2012
To support internal assessment from 2013

This task involves students investigating the trigonometric models representing two different Ferris wheels.

	Grade Boundary: Low Excellence
1.	For Excellence the student is required to apply trigonometric methods, using extended abstract thinking, in solving problems. This involves devising a strategy to investigate or solve a problem and using correct mathematical statements or communicating mathematical insight. There is evidence of extended abstract thinking in finding a reasonable (consistent) interval in solving the problem (1) and showing an understanding of a general solution (2). For a more secure Excellence the student would need to have found the correct equation for the Flying-high wheel (3), and provided a formalised general solution. The student could also consider that one of the intervals is not a sensible solution and choose to discard it (with explanation) (4). It should be noted that the incorrect equation for Manu's wheel does not simplify the overall problem.

Math's End Ferris Wheels.

Jade - Kiddy Wheel.
   height   0.5m to 8m.
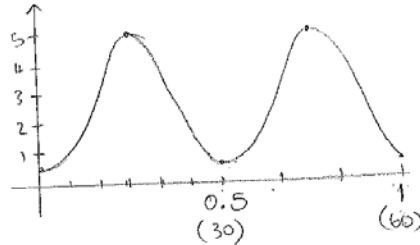   2 revolutions/minute

$8 - 0.5 = 7.5$
$7.5 \div 2 = 3.75$

$y = A \sin B (t - C) + D$.

$A = 3.75$
$D = 3.75 + 0.5 = 4.25$
$B = \frac{\pi}{15}$

$B = \frac{2\pi}{\frac{0.5}{30}} = \frac{\pi}{\frac{0.25}{15}}$

So by process of elimination
Kiddy Wheel is $h(t) = 3.75 \sin \frac{\pi}{15} (t - 7.5) + 4.25$ ✓

Manu - Flying-High Ferris
   height   3m to 43m
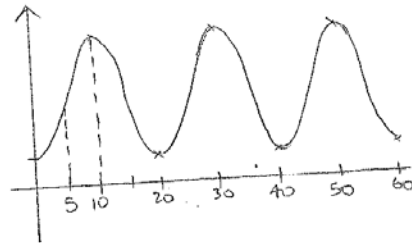   3 revolutions/minute.

$43 - 3 = 40.$
$40 \div 2 = 20.$
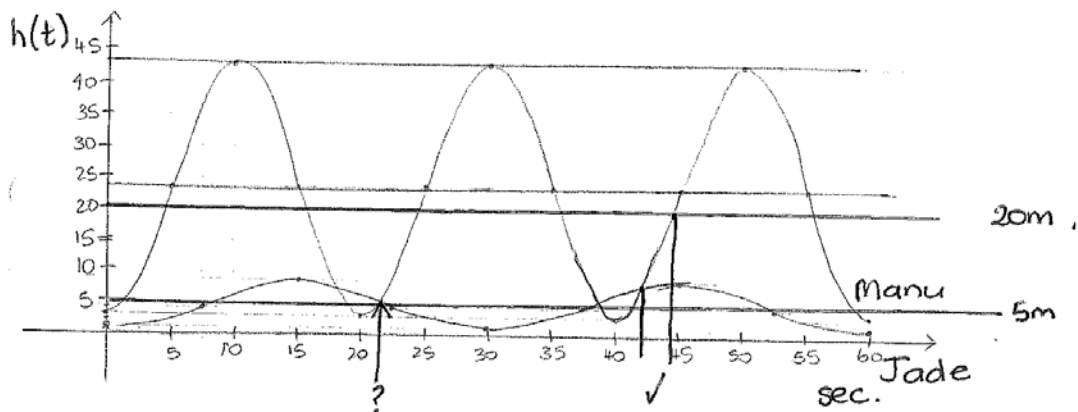
$A = 20$
$D = 23$
$B = \frac{\pi}{10}$.
$C = 5$

$B = \frac{2\pi}{20} = \frac{\pi}{10}.$ $C = 5$

So. $h(t) = 20 \sin \frac{\pi}{10} (t - 5) + 23$ ③

looking at the two graphs together.

$h(t)$

where can Jade see Manu.

Jade ≥ 5

$$3.75 \sin \frac{\pi}{15}(t-7.5) + 4.25 \geq 5$$

t is between 8.461 & 21.5385   G.C.
and 38.461 & 51.5385

Manu - going up, ≥ 5 and ≤ 20.

$$5 \leq 20 \sin \frac{\pi}{10}(t-5) + 23 \leq 20$$

t is between 1.44 & 4.52
21.44 & 24.52   G.C.
and. 41.44 & 44.52

so the intersection of these solutions is the time Jade can see Manu in the first 60 secs

(1) t is between 21.44 to 21.54 sec   (4)
and   41.44 to 44.52 sec

this will happen every 60 seconds for the duration of the ride. (2)

	Grade Boundary: High Merit
2.	For Merit the student is required to apply trigonometric methods, using relational thinking, in solving problems. This involves forming and using a model and relating findings to a context or communicating thinking using appropriate mathematical statements. The student has demonstrated relational thinking in linking the intervals found from the two correct equations and has communicated this in context (1). In investigating Manu's position on the Kiddy-wheel the student has considered the times when Manu is above 5m and below 20m, but not the times when Manu is also going up (2). To reach Excellence the student needs to discuss the abstract thinking required for generalising how the interval would continue beyond 2 minutes, and to consider that Manu can only be seen when going up.

Kiddy wheel

Max 8m, Min ramp 0.5
Amplitude 3.75
Translated up 4.25
Period = 30 $f = \dfrac{2\pi}{30}$

                     $= \dfrac{\pi}{15}$

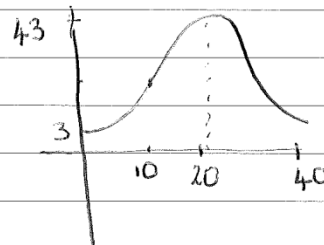Jade $h(t) = 3.75 \sin \dfrac{\pi}{15}(t - 7.5) + 4.25$ ①

Flying high

Max 43, Min 3
Amp 20
Translated up 23

3 Rev in 2min
Period 40 $f = \dfrac{2\pi}{40}$

                    $= \dfrac{\pi}{20}$

Manu $h(t) = 20 \sin \dfrac{\pi}{20}(t - 10) + 23$ ①

Jade   above 5m
         $5 = 3.75 \sin \dfrac{\pi}{15}(t - 7.5) + 4.25$
         $0.2 = \sin \dfrac{\pi}{15}(t - 7.5)$

         $0.2013 = \dfrac{\pi}{15}(t - 7.5)$

         $t = 8.46, \quad 21.54, \quad 38.46, \quad 51.54$

Manu   above 5   and below 20 ②

$$20 = 20 \sin \frac{\pi}{20}(t - 10) + 23$$

$$\frac{-3}{20} = \sin \frac{\pi}{20}(t - 10)$$

$$-0.1506 = \frac{\pi}{20}(t - 10)$$

$$t = 9.04, \quad 30.96, \quad 49.04, \quad 70.96, \quad 89.04, \quad 110.96.$$

$$5 = 20 \sin \frac{\pi}{20}(t - 10) + 23.$$

$$-1.1197 = \frac{\pi}{20}(t - 10)$$

$$t = 2.87, \quad 37.13, \quad 42.87, \quad 77.13, \quad 82.89, \quad 117.13 \ldots$$

Manu can be seen
2.87 - 9.04 , 30.96 - 37.13 , 42.87 - 49.04
70.96 - 77.13 , 82.87 - 89.04 , 110.96 - 117.13

Jade can be seen
8.46 - 21.54 , 38.46 - 51.54 , 68.46 - 81.54
98.46 - 111.54

Can see each other
8.46 - 9.04 sec , 42.87 - 49.04
70.96 - 77.13 , 110.96 - 111.54

1

y=-20cos(πx/20)+23
y=3.75sin((π/15)(x-7.5))+4.25
y=5
y=20

	Grade Boundary: Low Merit
3.	For Merit the student is required to apply trigonometric methods, using relational thinking, in solving problems. This involves forming and using a model and relating findings to a context or communicating thinking using appropriate mathematical statements. There is evidence of relational thinking with correctly forming the model of Manu's wheel (1) and using the model to determine an appropriate time interval for when Manu can be seen (2). The student has shown contextual understanding by connecting the times when Jade is above 5m and finding a realistic interval (3). For a more secure Merit the student needs both models to be correct with a correct interval. The incorrect equation for Jade's wheel does not over simplify the problem that has been solved.

Jade's wheel

$h(t) = 4 \cos \frac{\pi}{30}(t - 15) + 4.25$

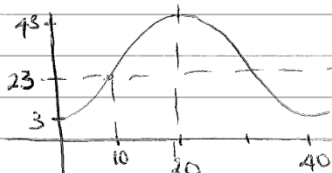amplitude $= \frac{8}{2} = 4$ period 30 sec   moves up

Memu's wheel.

$h(t) = -20 \cos \frac{\pi}{20} t + 23$ ①

amplitude $\frac{43 - 3}{2} = 20$ period   3 revs $= 120$ secs
                                              1 rev $= 40$ sec
                                              $\frac{2\pi}{40} = \frac{\pi}{20}$

Move up 23

Jade's wheel above 5m

  $1.801 \longrightarrow 28.198$ sec   , $61.8 \longrightarrow 88.2$ sec

Memu's wheel   above 5m and below 20m ②
  $2.87 \longrightarrow 9.04$ sec   , $82.87 \longrightarrow 89.04$ sec

Can see each other   between 2.87 and 9.04 seconds

③

	Grade Boundary: High Achieved
4.	For Achieved the student is required to apply trigonometric methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. The student has applied trigonometric methods in solving problems. There is evidence of methods related to features and equations of trigonometric functions in the students' examination of the periods of the possible models to find the correct equation for the Kiddy-wheel (1), and in finding the correct equation for the Flying-high wheel with evidence of an understanding of amplitude, period and vertical movement shown on the graph (2). The student has applied trigonometric methods in solving equations to find correct intervals for both Ferris wheels (3). To provide evidence of relational thinking and reach Merit the student needs to link the solutions back to the problem to find an interval when Jade can see Manu, and show a contextual understanding of the problem.

Jade. 1. $h(t) = 3.75 \sin \pi t + 4.5$

drew on G.C.

60 rotations in 60 seconds - unbelievable.

2. $h(t) = 3.75 \sin \frac{\pi}{15}(t - 7.5) + 4.25$

drew on G.C.

looks good.

3. $h(t) = 4 \cos \frac{\pi}{30}(t - 15) + 4.25$

drew on G.C.
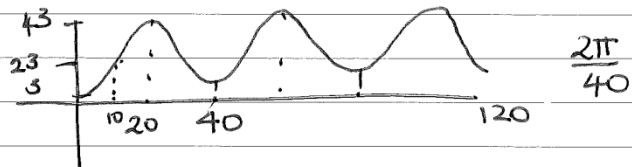
- period not correct

4. $h(t) = -4 \cos \pi t + 4.5$

drew on G.C.

60 rotations. - unbelievable.

(1) So $\Rightarrow$ $h(t) = 3.75 \sin \frac{\pi}{15}(t - 7.5) + 4.5$

is the equation.

Manu:

(2)

$h(t) = 20 \sin \frac{\pi}{20}(t - 10) + 23.$

Jade: $3.75 \sin \frac{\pi}{15}(t - 7.5) + 4.25 \geqslant 5$

(3) $8.461 \leq t \leq 21.5385 \; , \quad 38.461 \leq t \leq 51.5385$

Manu: $5 \leq 20 \sin \frac{\pi}{20}(t - 10) + 23 \leq 20$, going up.

$2.87 \leq t \leq 9.04, \; 42.87 \leq t \leq 49.04, \; 82.87 \leq t \leq 89.04$

	Grade Boundary: Low Achieved
5.	For Achieved the student is required to apply trigonometric methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. The student has applied trigonometric methods related to features of trigonometric functions to give the equation for the Kiddy-wheel, with evidence of understanding the features shown by notation on the graph linked to the equation (1). The student has solved a trigonometric equation to get an interval when Jade is above 5m (2). For a more secure Achieved the equation for the Flying-high wheel would be complete, and an interval for Manu found.

$A = 3.75$

$D = 4.25$

(15)
B
30

60

Here is the
cycle length
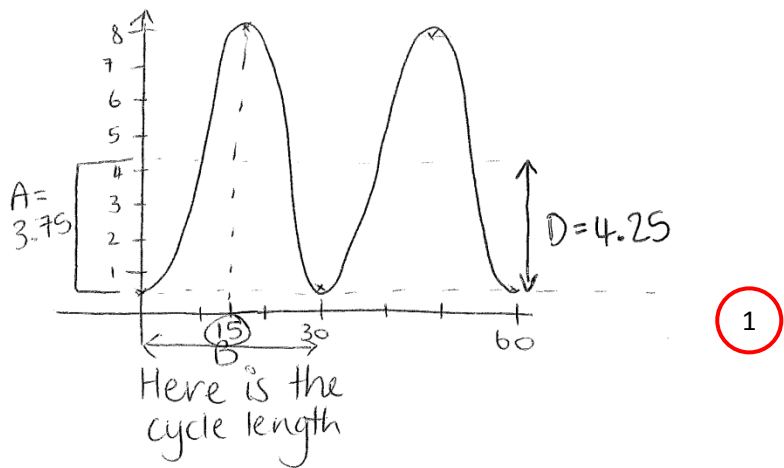
Kiddy wheel: $h = 3.75 \sin \frac{\pi}{15}(t - 7.5) + 4.25$
$\;A\phantom{.75 \sin \frac{\pi}{15}}BC_{??}D$

$(43 - 3) \div 2$

Flying high: $h = 20 \sin B(t - c) + 23$
moved up 23
$(3 + 20)$

B: 3 revolutions in 2 minutes ?
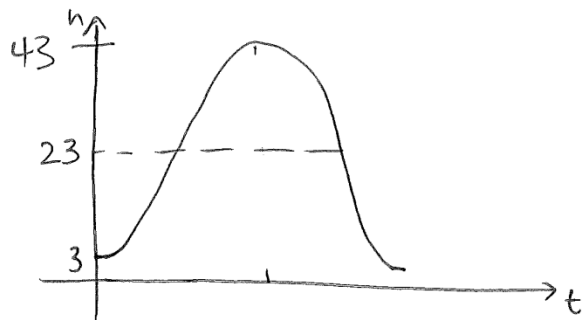
Jade > 5m   t between 8.46 and 21.54 (G.C.)

Manu < 20m and going up...

	Grade Boundary: High Not Achieved
6.	To achieve the standard the student is required to apply trigonometric methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations.
	The student has identified features of the trigonometric model for the Kiddy-wheel to get a correct model for the Ferris wheel (1).
	While a time when the Kiddy-wheel is 5m high has been found, in order to reach Achieved the student needs to apply the solution of the equation to determine a time interval when Jade is 5m or more above the ground (2).

Jade – Kiddy wheel.

amplitude 3.75
moved up 4.25 $h(t) = 3.75 \sin \frac{\pi}{15}(t - 7.5) + 4.25$
period   30sec

①

Jade at 5m
$5 = 3.75 \sin \frac{\pi}{15}(t - 7.5) + 4.25$

②

$t = 8.46.$