National Certificate of Educational Achievement
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Exemplar for Internal Assessment Resource

Mathematics Level 3

Resource title: Exact Values

This exemplar supports assessment against:

Achievement Standard 91575

Apply trigonometric methods in solving problems

Student and grade boundary specific exemplar
The material has been gathered from student material specific to an A or B assessment resource.

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	Grade Boundary: Low Excellence
1.	For Excellence the student is required to apply trigonometric methods, using extended abstract thinking, in solving problems. This involves devising a strategy to investigate or solve a problem and using correct mathematical statements or communicating mathematical insight. There is evidence of extended abstract thinking in using the periodic nature of the trigonometric functions to identify other angles that can be found from an exact value, for example "*tan(22.5 + 180n)"* for other angles with the same value as tan22.5 and using the double angle formulae for cos2A to find a half angle (1). For a more secure Excellence the student would have written a report to communicate their findings. They could also investigate generalisations for other exact values and derive the half angle formulae for any angle.

Student 1: Low Excellence

$\sin 60 = \frac{\sqrt{3}}{2}$ so $\operatorname{cosec} 60 = \frac{2}{\sqrt{3}}$

$\cos 60 = \frac{1}{2}$ so $\sec 60 = 2$

$\tan 60 = \sqrt{3}$ so $\cot 60 = \frac{1}{\sqrt{3}}$

$\sin 45 = \frac{1}{\sqrt{2}}$ so $\operatorname{cosec} 45 = \sqrt{2}$

$\cos 45 = \frac{1}{\sqrt{2}}$ so $\sec 45 = \sqrt{2}$

$\tan 45 = 1$ so $\cot 45 = 1$

$\cos 22.5 = \cos \frac{45}{2}$

$A = 22.5$, $\cos 2A = 2\cos^2 A - 1 \rightarrow \cos 45 = 2\cos^2 \frac{45}{2} - 1$

$\cos 45 = \frac{1}{\sqrt{2}}$ so $\frac{1}{\sqrt{2}} + 1 = 2\cos^2 \frac{45}{2}$

$\frac{1+\sqrt{2}}{2\sqrt{2}} = \cos^2 \frac{45}{2}$

$\cos 22.5 = \frac{\sqrt{1+\sqrt{2}}}{\sqrt{2\sqrt{2}}}$ 1

then $\sin 2A = 2 \sin A \cos A$ so $\sin A = \frac{\sin 2A}{2\cos A}$

so $\sin 22.5 = \frac{\sin 45}{2\cos 22.5} = \frac{\sin 45}{2} \times \frac{1}{\cos 22.5}$

$= \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2\sqrt{2}}}{\sqrt{1+\sqrt{2}}} = \frac{1}{(\sqrt{2\sqrt{2}})(\sqrt{1+\sqrt{2}})}$

$\tan^2 A = \sec^2 A - 1$

$\tan^2 22.5 = \left(\frac{\sqrt{2\sqrt{2}}}{\sqrt{1+\sqrt{2}}}\right)^2 - 1 = \frac{2\sqrt{2}}{1+\sqrt{2}} - \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{2\sqrt{2} - 1 - \sqrt{2}}{1+\sqrt{2}}$

$= \frac{\sqrt{2} - 1}{1+\sqrt{2}}$

$\tan 22.5 = \frac{\sqrt{-1+\sqrt{2}}}{\sqrt{1+\sqrt{2}}}$ $\tan(22.5 + 180n) = \frac{\sqrt{-1+\sqrt{2}}}{\sqrt{1+\sqrt{2}}}$

$\sin 105 = \sin(60+45) = \sin 60 \cos 45 + \cos 60 \sin 45$

$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\cos 105 = \cos(60+45) = \cos 60 \cos 45 - \sin 60 \sin 45$

$= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

$\tan 105 = \tan(60+45) = \frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45} = \frac{\sqrt{3}+1}{1-\sqrt{3}} = \frac{1+\sqrt{3}}{1-\sqrt{3}}$

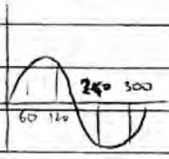
$\sin 15 = \sin(60-45) = \sin 60 \cos 45 - \cos 60 \sin 45$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\cos 15 = \sin(60-45) = \cos 60 \cos 45 + \sin 60 \sin 45$

$= \frac{1+\sqrt{3}}{2\sqrt{2}} = \sin 105$

$\sin 120 = \sin 60 = \frac{\sqrt{3}}{2}$ [illegible] $\cos(-60) = \cos 60 = \frac{1}{2}$



$\sin 240 = -\sin 60 = \frac{-\sqrt{3}}{2}$

$\sin 300 = \sin 240 = \frac{-\sqrt{3}}{2}$

$\sin 165 = \sin 15 = \frac{\sqrt{3}-1}{2\sqrt{2}}$ $\sin 195 = -\sin 15 = \frac{1-\sqrt{3}}{2\sqrt{2}}$

$\sin 345 = \sin 195 = \frac{1-\sqrt{3}}{2\sqrt{2}}$

$\sin(60+360n) = \frac{\sqrt{3}}{2}$ $\sin(120+360n) = \frac{\sqrt{3}}{2}$

$\sin(240+360n) = \frac{-\sqrt{3}}{2}$

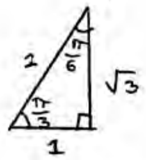
$\cos(\pm 60 + 360n) = \frac{1}{2}$ $\tan(60+180n) = \sqrt{3}$

	Grade Boundary: High Merit
2.	For Merit the student is required to apply trigonometric methods, using relational thinking, in solving problems. This involves forming and using a model and relating findings to a context or communicating thinking using appropriate mathematical statements. The student has demonstrated relational thinking in finding the reciprocal trigonometric functions for the angles found using the compound angle formulae and double angle formulae (1). The values for tan120° and cot120° are incorrect (2). In finding general solutions the student has found all the angles that have a sine and cosine of $1/\sqrt{2}$ and a tangent of 1, but has not clearly communicated what the angles represent (3). The statements for the reciprocal ratios are incorrect (4). To reach Excellence the generalisations need to be extended to exact values, other than those for the angles in the special triangles.

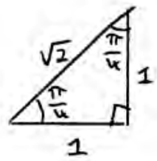
Student 2: High Merit

Special angles:

using;



and



I can determine that;

	sin	cos	tan
$30°, \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45°, \frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60°, \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{1}$

By using $\sin = \frac{O}{H}$

$\cos = \frac{A}{H}$

$\tan = \frac{O}{A}$

Therefore:

$\cot 30° = \frac{\sqrt{3}}{1}$ Because $\cot\theta = \frac{1}{\tan\theta}$

$\cot 45° = 1$

$\cot 60° = \frac{1}{\sqrt{3}}$

$\sec 30° = \frac{2}{\sqrt{3}}$ Because $\sec\theta = \frac{1}{\cos\theta}$

$\sec 45° = \frac{\sqrt{2}}{1}$

$\sec 60° = \frac{2}{1}$

$\operatorname{cosec} 30° = \frac{2}{1}$ Because $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$\operatorname{cosec} 45° = \frac{\sqrt{2}}{1}$

$\operatorname{cosec} 60° = \frac{2}{\sqrt{3}}$

$15°, \frac{\pi}{12}$

Compound Angles

$$\sin(60-45) = \sin 60 \cos 45 - \cos 60 \sin 45$$
$$= \left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2} \times \frac{1}{\sqrt{2}}\right)$$
$$= \left(\frac{\sqrt{3}}{2\sqrt{2}}\right) - \left(\frac{1}{2\sqrt{2}}\right)$$

$$\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}} \quad \therefore \operatorname{cosec} 15° = \frac{2\sqrt{2}}{\sqrt{3}-1}$$

$$\cos(60-45) = \cos 60 \cos 45 + \sin 60 \sin 45$$
$$= \left(\frac{1}{2} \times \frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}\right)$$
$$= \left(\frac{1}{2\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2\sqrt{2}}\right)$$

$$\cos 15° = \frac{1+\sqrt{3}}{2\sqrt{2}} \quad \therefore \sec 15° = \frac{2\sqrt{2}}{1+\sqrt{3}}$$

$$\tan(60-45) = \frac{\frac{\sqrt{3}}{1} - 1}{1 + \left(\frac{\sqrt{3}}{1} \times 1\right)}$$
$$= \frac{\frac{\sqrt{3}}{1} - 1}{\frac{\sqrt{3}}{1} + 1}$$

$$\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1} \quad \therefore \cot 15° = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Double ~~Ann~~ Angles

$120, \frac{2\pi}{3}$

$\sin 2A = 2\sin A\cos A$

$A = 60^\circ$

~~sin~~

$\sin 2(60^\circ) = 2\left(\frac{\sqrt{3}}{2} \times \frac{1}{2}\right)$

$= 2\left(\frac{\sqrt{3}}{2\sqrt{2}}\right)$

$\sin 120^\circ = \frac{2\sqrt{3}}{4\sqrt{2}}$ $\therefore$ $\operatorname{cosec} 120^\circ = \frac{4\sqrt{2}}{2\sqrt{3}}$

$\cancel{\cos 2A = \cos^2 A - \sin^2 A}$

$\cancel{A = 60^\circ}$

$\cancel{\cos 2(60^\circ) = (\cos 60^\circ)^2 - (\sin 60^\circ)^2}$

$\cancel{= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2}$

$\cancel{= \frac{1}{4} - \frac{3}{2}}$

$\cancel{= \frac{1}{4} - \frac{5}{4}}$

$\cancel{= -\frac{5}{4}}$

$\cos 2A = 2\cos^2 A - 1$

$A = 60^\circ$

$\cos 2(60^\circ) = 2\left(\frac{1}{2}\right)^2 - 1$

$= \frac{1}{2} - 1$

$\cos 120^\circ = -0.5$ 1

$\therefore \sec 120^\circ = \frac{1}{-0.5}$

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

$A = 60^\circ$

$\tan 2(60^\circ) = \frac{2\left(\frac{\sqrt{3}}{1}\right)}{1-\left(\frac{\sqrt{3}}{1}\right)^2}$

$\therefore \cot 120^\circ = \frac{-2}{2\sqrt{3}}$ $= \frac{\frac{2\sqrt{3}}{2}}{1-3}$ $\tan 120^\circ = \frac{\frac{2\sqrt{3}}{2}}{-2}$

2

General solutions

$\cancel{\sin 45^\circ = \frac{1}{\sqrt{2}}}$

$\cancel{45^\circ = \sin^{-1}\frac{1}{\sqrt{2}}}$

$\sin x = \frac{1}{\sqrt{2}}$

$x = \sin^{-1}\frac{1}{\sqrt{2}}$

$x = 45^\circ$

$\theta = n180^\circ + (-1)^n 45^\circ$

~~Ans~~

$\cos x = \frac{1}{\sqrt{2}}$

$x = \cos^{-1}\frac{1}{\sqrt{2}}$

$x = 45^\circ$

~~θ ...~~

$\theta = 2(180^\circ)n \pm 45^\circ$ 3

$\tan x = 1$

$x = \tan^{-1} 1$

$x = 45^\circ$

$\theta = n180^\circ + 45^\circ$

Therefore:

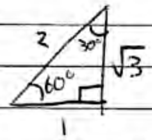
~~cosec~~

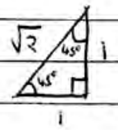
$\operatorname{cosec} x = n\frac{1}{180} + (-1)^n \frac{1}{45}$

$\sec x =$ ~~Ans~~ $n\left(\frac{1}{360}\right) \pm \frac{1}{45^\circ}$ 4

$\cot x = n\frac{1}{180} + \frac{1}{45}$

	Grade Boundary: Low Merit
3.	For Merit the student is required to apply trigonometric methods, using relational thinking, in solving problems. This involves forming and using a model and relating findings to a context or communicating thinking using appropriate mathematical statements. There is evidence of relational thinking in finding the reciprocal trigonometric functions for the angles found using the compound angle formulae (1). For a more secure Merit the student could use the unit circle or graphs to investigate other angles.





deg	rad
$30° =$	$\frac{\pi}{6}$
$90° =$	$\frac{\pi}{2}$
$45° =$	$\frac{\pi}{4}$
$60° =$	$\frac{\pi}{3}$

$\sin\theta = \frac{O}{H}$ $\quad \sin 30 = \frac{1}{2}$ $\quad \sin 45 = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{A}{H}$ $\quad \cos 30 = \frac{\sqrt{3}}{2}$ $\quad \cos 45 = \frac{1}{\sqrt{2}}$

$\tan\theta = \frac{O}{A}$ $\quad \tan 30 = \frac{1}{\sqrt{3}}$ $\quad \tan 45 = \frac{1}{1} = 1$

$\sin 60 = \frac{\sqrt{3}}{2}$

$\cos 60 = \frac{1}{2}$

$\tan 60 = \frac{\sqrt{3}}{1} = \sqrt{3}$

$\operatorname{cosec}\theta = \frac{H}{O}$ $\quad \operatorname{cosec} 30 = \frac{2}{1} = 2$ $\quad \operatorname{cosec} 45 = \frac{\sqrt{2}}{1} = \sqrt{2}$

$\sec\theta = \frac{H}{A}$ $\quad \sec 30 = \frac{2}{\sqrt{3}}$ $\quad \sec 45 = \frac{\sqrt{2}}{1} = \sqrt{2}$

$\cot\theta = \frac{A}{O}$ $\quad \tan 30 = \frac{\sqrt{3}}{1} = \sqrt{3}$ $\quad \cot 45 = 1$

$\operatorname{cosec} 60 = \frac{2}{\sqrt{3}}$

$\sec 60 = \frac{2}{1} = 2$

$\cot 60 = \frac{1}{\sqrt{3}}$

$$\sin 75 = \sin(45 + 30) = \sin 45 \cos 30 + \cos 45 \sin 30$$
$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$
$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\sin 105 = \sin(45 + 60) = \sin 45 \cos 60 + \cos 45 \sin 60$$
$$= \frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$$
$$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$
$$= \frac{1+\sqrt{3}}{2\sqrt{2}}$$

$$\sin 90 = \sin(30 + 60) = \sin 30 \cos 60 + \cos 30 \sin 60$$
$$\frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = 1$$

$\cos 75 = \cos(45+30) = \cos 45 \cos 30 - \sin 45 \sin 30$

$\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\cos 105 = \cos(45+60) = \cos 45 \cos 60 - \sin 45 \sin 60$

$\frac{1}{\sqrt{2}} \times \frac{1}{2} - \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$

$\frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} = \frac{1-\sqrt{3}}{2\sqrt{2}}$

$\cos 90 = \cos(30+60) = \cos 30 \cos 60 - \sin 30 \sin 60$

$\frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

$\tan 75 = \tan(45+30) = \frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$

$\tan 105 = \tan(45+60) = \frac{\tan 45 + \tan 60}{1 - \tan 45 \tan 60} = \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$

$\tan 90 = \tan(30+60) = \frac{\tan 30 + \tan 60}{1 - \tan 30 \tan 60} = \frac{\sqrt{3} + \frac{1}{\sqrt{3}}}{1 - 1} = \frac{\sqrt{3} + \frac{1}{\sqrt{3}}}{0}$ = undefined.

$\cancel{\sin 15 = \sin(45-30) = \sin 45 \cos 30 - \cos 45 \sin 30}$

$\cancel{\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}}$

$\cancel{\frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2}}$

$\sin 15 = \sin(60-45) = \sin 60 \cos 45 - \cos 60 \sin 45$

$\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\cos 15 = \cos(45-30) = \cos 45 \cos 30 + \sin 45 \sin 30$

$\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}}$

$\tan 15 = \tan(45-30) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$

$\operatorname{Cosec} 75 = \frac{2\sqrt{2}}{\sqrt{3}+1}$

$\operatorname{Cosec} 105 = \frac{2\sqrt{2}}{1+\sqrt{3}}$

$\operatorname{Cosec} 90 = 1$

$\operatorname{Sec} 75 = \frac{2\sqrt{2}}{\sqrt{3}-1}$

$\operatorname{Sec} 105 = \frac{2\sqrt{2}}{1-\sqrt{3}}$

$\operatorname{Sec} 90 = 0$

$\operatorname{Cot} 75 = \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$

$\operatorname{Cot} 105 = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

Cot 90 = unidentified

$\operatorname{cosec} 15 = \frac{2\sqrt{2}}{\sqrt{3}-1}$

$\operatorname{Sec} 15 = \frac{2\sqrt{2}}{1+\sqrt{3}}$

$\operatorname{Cot} 15 = \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$ 1

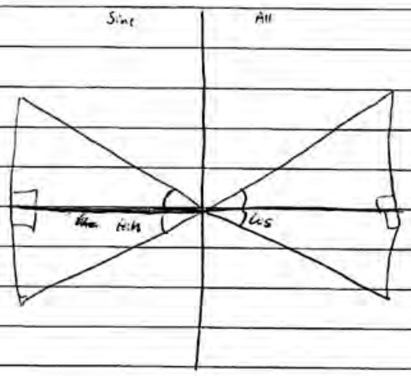
	Grade Boundary: High Achieved
4.	For Achieved the student is required to apply trigonometric methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. The student has applied trigonometric methods in solving problems in selecting and using the unit circle to determine exact values for angles beyond the first quadrant, compound angle formulae and reciprocal trigonometric functions (1). The correct value for cos105° has been found using incorrect working (2). The student has attempted to use the double angle formula with the exact values for 105° found using the compound angle formulae, but the value given for sin210° is incorrect because there is another error with the value for cos105° (3). To reach Merit the student needs to correctly connect different concepts and representations. For example, the student could find exact values for other angles using the compound angle formulae and then find the reciprocal trigonometric functions for these angles, or extend the use of the unit circle to compound angles.

Student 4: High Achieved

$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

$\tan 60 = \frac{1}{\sqrt{3}}$



$\sin 120 = \frac{\sqrt{3}}{2}$

$\cos 120 = \frac{-1}{2}$

$\sin 240 = -\frac{\sqrt{3}}{2}$

$\cos 240 = \frac{-1}{2}$

$\sin 300 = -\frac{\sqrt{3}}{2}$

$\cos 300 = \frac{1}{2}$

To find the exact values of these angles I have used

$\sin 420 = \frac{\sqrt{3}}{2}$

$\cos 420 = \frac{1}{2}$ 1

Compound angles

$\sin(45 + 60) = \sin 45 \cos 60 + \cos 45 \sin 60$

$= \frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{2\sqrt{2}} + \frac{1\sqrt{3}}{2\sqrt{2}}$

$\sin 105 = \frac{1+\sqrt{3}}{2\sqrt{2}}$ 1

~~$\cos(45+60) = \cos 45 \cos 60 + \sin 45 \sin 60$~~

~~$= \frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$~~

~~$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$~~

~~$= \frac{\sqrt{3}+1}{2\sqrt{2}}$~~

~~Double Angles~~

~~$\sin 2A = 2\sin A \cos A$~~

~~$\sin 210 = 2\sin 105 \cos 105$~~

~~$= 2 \frac{1+\sqrt{3}}{2\sqrt{2}} \times$~~

$\cos(45 + 60) = \cos 45 \cos 60 - \sin 45 \sin 60$

$= \frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$ 2

$= \frac{-\sqrt{3}+1}{2\sqrt{2}}$

Negative because it is in the sine area of the graph.

~~$\sin(300 - 60)$~~

$\sin 2(105) = 2\sin 105 \cos 105$

$\sin 210 = 2 \frac{1+\sqrt{3}}{2\sqrt{2}} \times \frac{-1+\sqrt{3}}{2\sqrt{2}}$

$= \frac{1+2\sqrt{3}}{2\sqrt{2}} \times \frac{-1+\sqrt{3}}{2\sqrt{2}}$ 3

$= \frac{-1+\sqrt{3}-2\sqrt{3}+6}{8}$

$= \frac{5-\sqrt{3}}{8}$

$$\operatorname{cosec} 60 = \frac{1}{\sin 60}$$

$$= \frac{1}{\frac{1}{2}\sqrt{3}}$$

$$= \cancel{\tfrac{\sqrt{3}}{2}} \quad \frac{1}{1/2} \times \frac{1}{\sqrt{3}}$$

$$= 2 \times \frac{1}{\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}}$$

$$\sec 60 = \frac{1}{\cos 60}$$

$$= 2$$

$$\cot 60 = \frac{1}{\tan 60}$$

$$= \frac{1}{\frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{1}{1}}{\sqrt{3}} \times \frac{\sqrt{3}}{1}$$

$$= \sqrt{3} \times 1$$

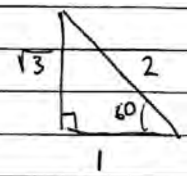
$$= \sqrt{3}$$

1

	Grade Boundary: Low Achieved
5.	For Achieved the student is required to apply trigonometric methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. The student has applied trigonometric methods in solving problems in selecting and using reciprocal trigonometric functions and the double angle formulae (1). For a more secure Achieved the student needs to work with more than one special angle and should also use other trigonometric methods.

Student 5: Low Achieved

~~\$~~ $\theta = n180 + (-1)^n 60°$



$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \cancel{\sqrt{3}}\ \frac{1}{2}$

$\sin 120° = 2\left(\frac{\sqrt{3}}{2} \times \frac{1}{2}\right)$

$\tan 60° = \sqrt{3}$

$\sin 120° = \frac{\sqrt{3}}{2}$

~~sin~~ $\cos 120° = \left(\frac{1}{2}\right)^2 - \cancel{\sin}\left(\frac{\sqrt{3}}{2}\right)^2$

$\cos 120° = 0.25 - 0.75$

$= -\frac{1}{2}$

$\tan 120° = \frac{2\tan 60}{1 - \sqrt{3}^2}$

$= \frac{2\sqrt{3}}{1-3}$

$= \frac{2\sqrt{3}}{-2}$

$= -\sqrt{3}$

~~cos~~ ~~cot~~ $\theta = 2n\pi \pm \frac{4}{\pi}$

$\cot 120° = -\frac{1}{\sqrt{3}}$

1

~~cosec 60 = 60°~~

$\operatorname{cosec} 60° = \frac{2}{\sqrt{3}}$

$\sec 60° = 2$

$\cot 60° = \frac{1}{\sqrt{3}}$

	Grade Boundary: High Not Achieved
6.	For Achieved the student is required to apply trigonometric methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. The student has selected and used reciprocal trigonometric functions to find exact values for some of the reciprocal functions for the angles in the special triangles (1). The use of the double angle formula to find sin90° is correct, but this is a known angle (2). While the statement of the compound angle formula is correct, to reach Achieved the student needs to determine the exact value for sin135°.

Student 6: High Not Achieved

$\sin 30° = \frac{1}{2}$ $\quad$ $\operatorname{cosec} 30° = 2$ $\quad$ ~~sin~~

$\cancel{\sin(2\times 30) = 2\sin 30 \times \cos 30}$ $\quad$ $\cos 60 = \frac{1}{2}$ $\quad$ $\sec 60 = 2$

$\sin 45° = \frac{1}{\sqrt{2}}$ $\quad$ $\operatorname{cosec} 45° = \sqrt{2}$

$\cos 45° = \frac{1}{\sqrt{2}}$ $\quad$ $\sec 45° = \sqrt{2}$ $\quad$ 1

$\sin(45\times 2) = 2\times \sin 45 \times \cos 45°$

$= 2\times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$ $\quad$ $\cancel{\sin(30+90) = \frac{1}{2}\times 0 + 1\times}$

$= 2\times \frac{1}{2}$

$\sin 90° = 1$ $\quad$ $\sin(90+45) = \sin 90 \times \cos 45 + 0 \times \sin 45$

$\cos 90° = 0$ $\quad$ 2 $\quad$ $\sin 135° =$