National Certificate of Educational Achievement
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Exemplar for Internal Assessment Resource

Mathematics Level 3

Resource title: Ted's Tomatoes

This exemplar supports assessment against:

Achievement Standard 91574

Apply linear programming methods in solving problems

Student and grade boundary specific exemplar

The material has been gathered from student material specific to an A or B assessment resource.

Date version published by Ministry of Education: December 2012
To support internal assessment from 2013

The task asks students to use the constraints provided to recommend the number of hectares of artichokes and tomatoes for Ted to maximize his income in the current year and in future years.

	Grade Boundary: Low Excellence
1.	For Excellence the student is required to apply linear programming methods, using extended abstract thinking, in solving problems. This involves devising a strategy to investigate or solve a problem, developing a chain of logical reasoning and using correct mathematical statements or communicating mathematical insight. There is evidence of extended and abstract thinking by providing the feasible region **(1)** to satisfy all the constraints and the student has made a recommendation regarding the number of hectares of artichokes and tomatoes required to maximise the income **(2)**. The student has identified that there are multiple solutions for this situation in future years **(3)**. For a more secure Excellence the student would need to accurately communicate that the multiple solutions all lie on the line formed by one of the constraints.

Equations

x = artichokes y = tomatoes

$20x + 10y \leq 1200$

$x + y \leq 90$

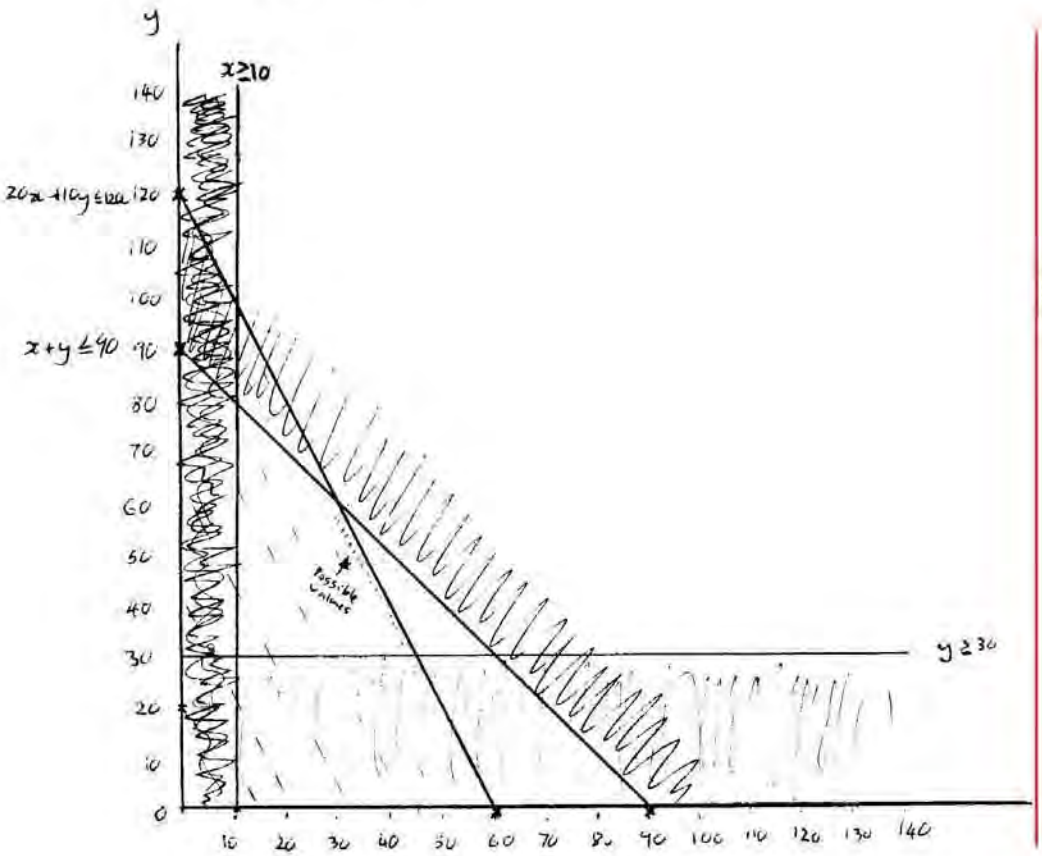
$y \geq 30$

$x \geq 10$

Constraint

$I = 25000x + 1000y$

Graph 1



1

Vertex	$25000x + 10000y$
(10, 30)	$ 550000
(10, 80)	$ 1050000
(30, 60)	$ 1350000
(45, 30)	$ 1425000

To maximise his income in the current year Ted should grow 45 hectares of artichokes and 30 hectares of tomatoes

2

For future income

Test values to show outcomes

eg If artichokes brought in $30000 per hectare and tomatoes brought in $15000 per hectare

Vertices	$30,000x + 15000y$
(10, 30)	$ 750000
(10, 80)	$ 1500000
(30, 60)	$ 1800000
(45, 30)	$ 1800000

In future there are multiple solutions to maximise profit

$30000x + 15000y$

$15000y = -30000x$

$\div 15000 \quad \div 15000$

$y = -2x$

Gradient $= \frac{-2}{1}$

Moving line approach

Intersects at line $20x + 10y \leq 1200$

3

Therefore Max profit in future can be achieved between $y = 30$ and 60 and $x = 10$ and 45

$30 \leq y \leq 60$

$10 \leq x \leq 45$

Therefore future max profit can be achieved if Ted grows between 30 and 60 hectares of tomatoes and if he grows between 10 and 45 hectares of artichokes

All working on Graph 1

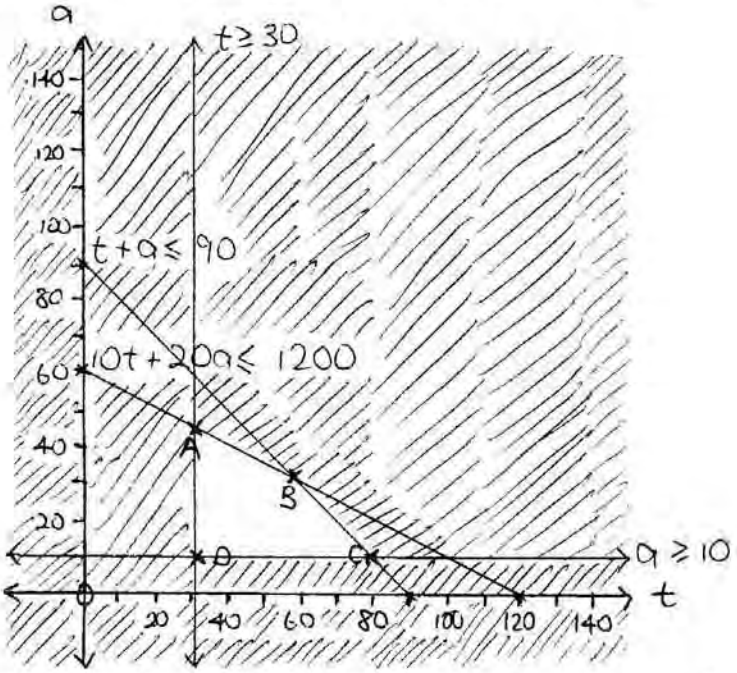
	Grade Boundary: High Merit
2.	For Merit the student is required to apply linear programming methods, using relational thinking, in solving problems. This involves selecting and carrying out a logical sequence of steps, connecting different concepts and representations, demonstrating understanding of concepts and also relating findings to a context, or communicating thinking using appropriate mathematical statements. The student has shown evidence of relational thinking by identifying the feasible region **(1)** for the system of linear inequalities and identifying the number of hectares required for each vegetable to maximise the income **(2)**. The student has identified that there are two solutions and has selected one as the optimal value for future years, but has not provided the evidence to support this claim **(3)**. To be awarded Excellence the student would need to identify that there are multiple solutions for the situation in future years based on supporting evidence.

t = tomatoes
a = artichokes

$t + a \leq 90$
$10t + 20a \leq 1200$
$t \geq 30$
$a \geq 10$

} equations

Income equation: $10,000t + 25,000a = I$



1

Each set of co-ordinates which are the vertices for the feasible region are put into the profit equation $I = 10000t + 25000a$

Vertices	$10000t + 25000a$	$10000t + 20000a$
A (30, 45)	1,425,000	
B (60, 30)	1,350,000	
C (80, 10)	1,050,000	
D (30, 10)	550,000	

In the current year, Ted should plant 30 hectares of tomatoes and 45 hectares of artichokes in order to maximise his income. If he does this, his income will be $1,425,000 according to his expectations regarding how much he will recieve per hectare.

2

Future payments of tomatoes : artichokes is predicted at 1:2
As the value was previously $10,000 per hectare of tomatoes and $25,000 per hectare of artichokes, the future value can be estimated as $10,000 for tomatoes and $20,000 for artichokes.
The new income equation would therefore be:

$$I = 10,000t + 20,000a$$

In future years, Ted could plant either 30 hectares of tomatoes and 45 hectares of artichokes, or 60 hectares of tomatoes and 30 hectares of artichokes, both options providing an income of $1,200,000.

3

However, seeing as artichokes are very labour-intensive, Ted's best option would be to plant 60 hectares of tomatoes and 30 hectares of artichokes in future years.

	Grade Boundary: Low Merit
3.	For Merit the student is required to apply linear programming methods, using relational thinking, in solving problems. This involves selecting and carrying out a logical sequence of steps, connecting different concepts and representations, demonstrating understanding of concepts and also relating findings to a context, or communicating thinking using appropriate mathematical statements. The student has shown evidence of relational thinking by providing the feasible region for the system of linear inequalities **(1)** and identifying the number of hectares for each vegetable to maximise the income for the current year **(2)**. For a more secure Merit the student would need to consider how the payments in the future would affect the optimal area for each vegetable.

Artichokes = x

Tomatoes = y

CONSTRAINTS:

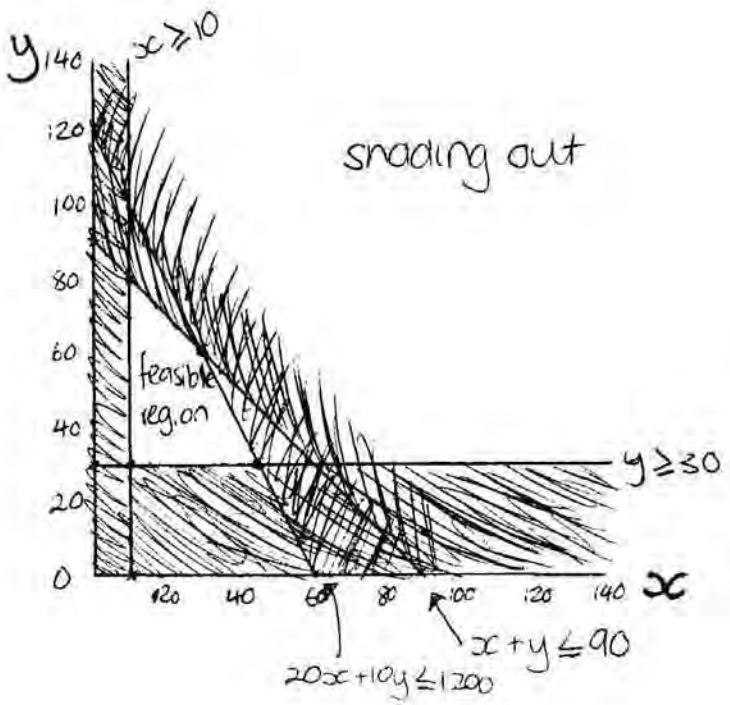
$x + y \leq 90$

$20x + 10y \leq 1200$

$x \geq 10$

$y \geq 30$

$I = 10\,000y + 25000x$



1

Vertices	$10\,000y + 25\,000x$
(10, 30)	$10,000(30) + 25,000(10) = 550,000$
(10, 80)	$10,000(80) + 25,000(10) = 1,050,000$
(45, 30)	$10,000(30) + 25,000(45) = 1,425,000$
(30, 60)	$10,000(60) + 25,000(30) = 1,350,000$

Maximum is with 45 hectares of artichokes and 30 of tomatoes.

2

	Grade Boundary: High Achieved
4.	For Achieved the student is required to apply linear programming methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. The student has shown evidence of applying linear programming methods by forming the constraints **(1)** and using them to identify the feasible region **(2)**. The student has also evaluated the income for each vertex of the feasible region **(3)**. To be awarded Merit the student would need to identify the vertex which maximises the income function to make a recommendation regarding the number of hectares for each vegetable.

A=x
T=y

$20A + 10T \leq 1200$

$A + T \leq 90$

$T \geq 30$

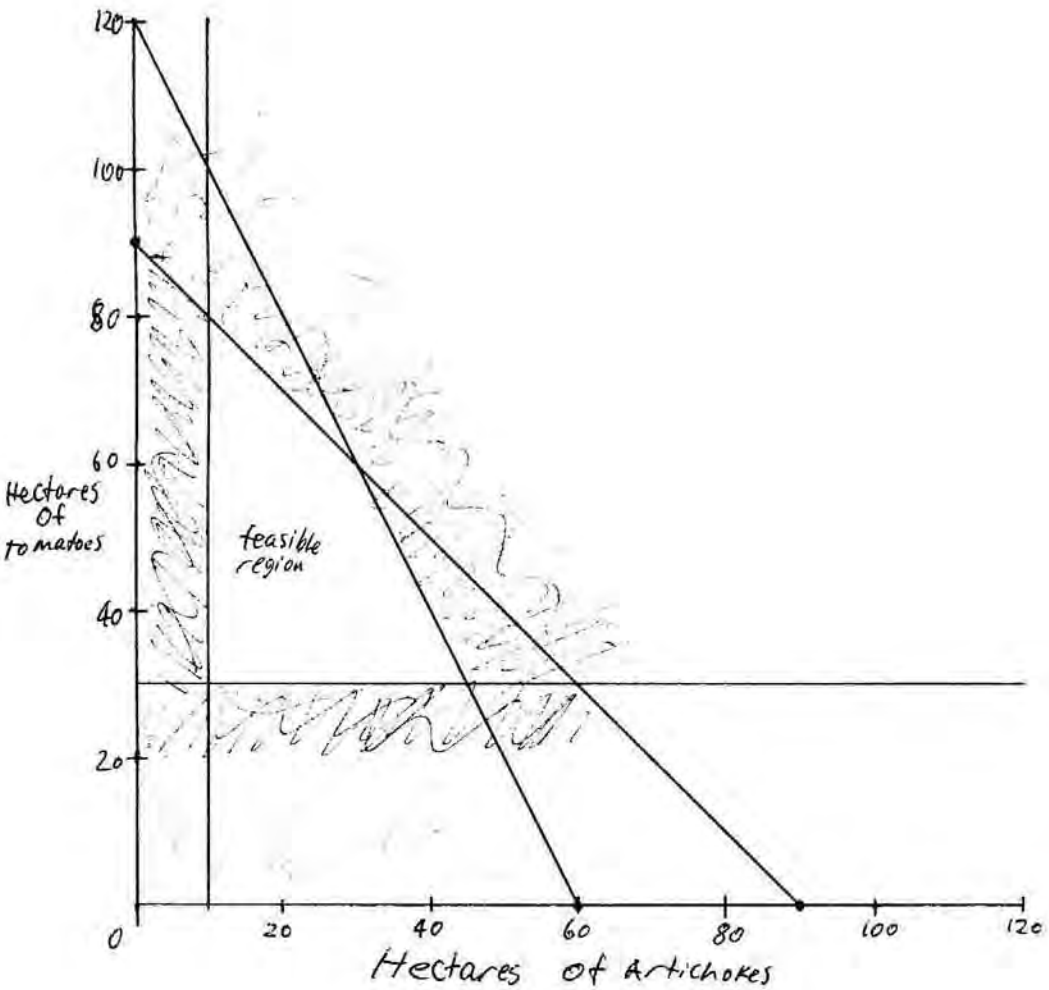
$A \geq 10$

1

$25{,}000A + 10{,}000T$

(30, 60)	$1,350,000
(45, 30)	$1,425,000
(10, 80)	$1,050,000
(10, 30)	$550,000

3



2

	Grade Boundary: Low Achieved
5.	For Achieved the student is required to apply linear programming methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. The student has shown evidence of linear programming methods by providing the equation of the inequality for the hours of labour **(1)** and the feasible region identified by the four constraints **(2)**. For a more secure Achieved the student would need to indicate what is represented by each variable.

Student 5: Low Achieved

equations

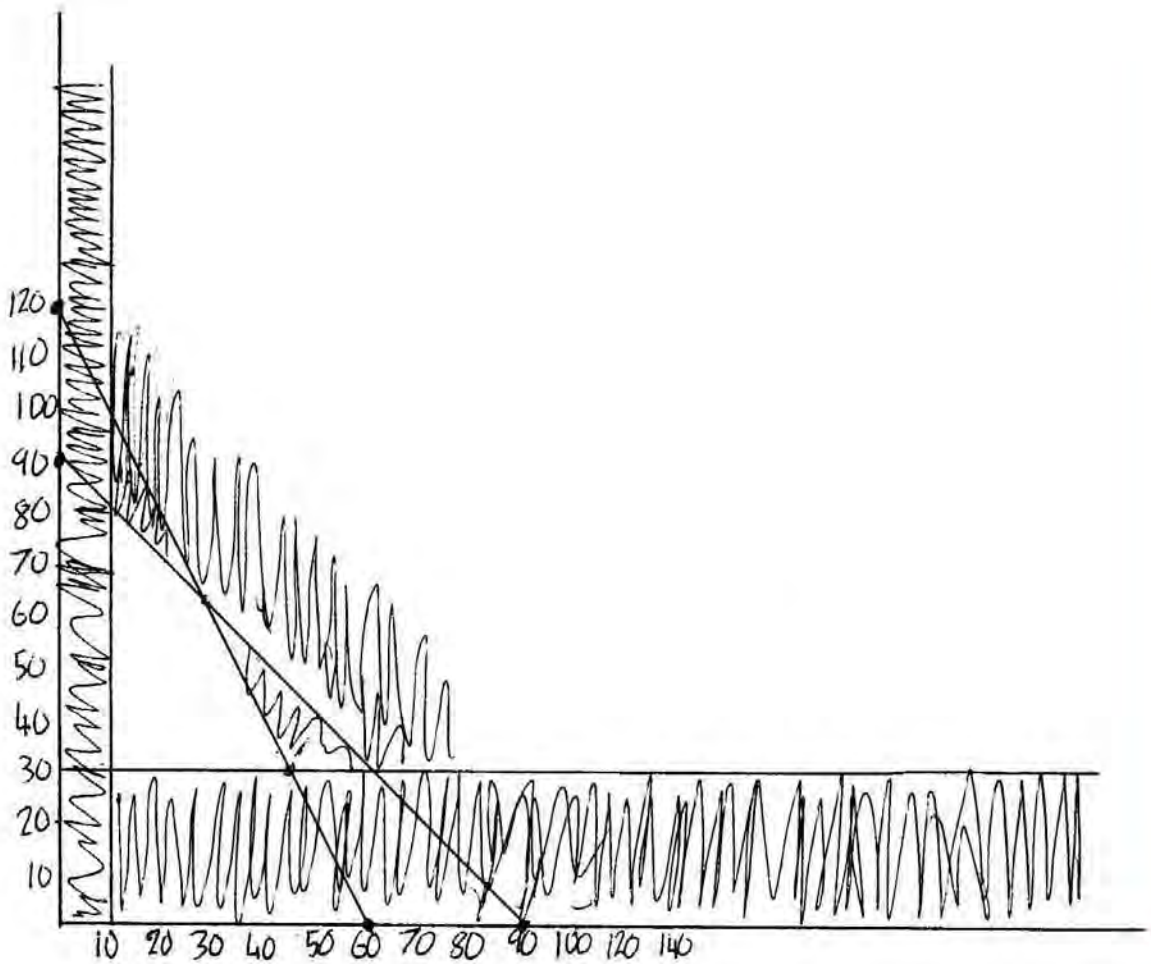
$20x + 10y \leq 1200$ (1)

$x + y \leq 90$

$y \geq 30$

$x \geq 10$

$I = 25\,000x + 10\,000y$



(2)

Intercepts : (10,30) (10,80) (30,60) (45,30)

Intercepts	25000x + 10000y
(10, 30)	
(10, 80)	
(30,60)	
(45,30)	

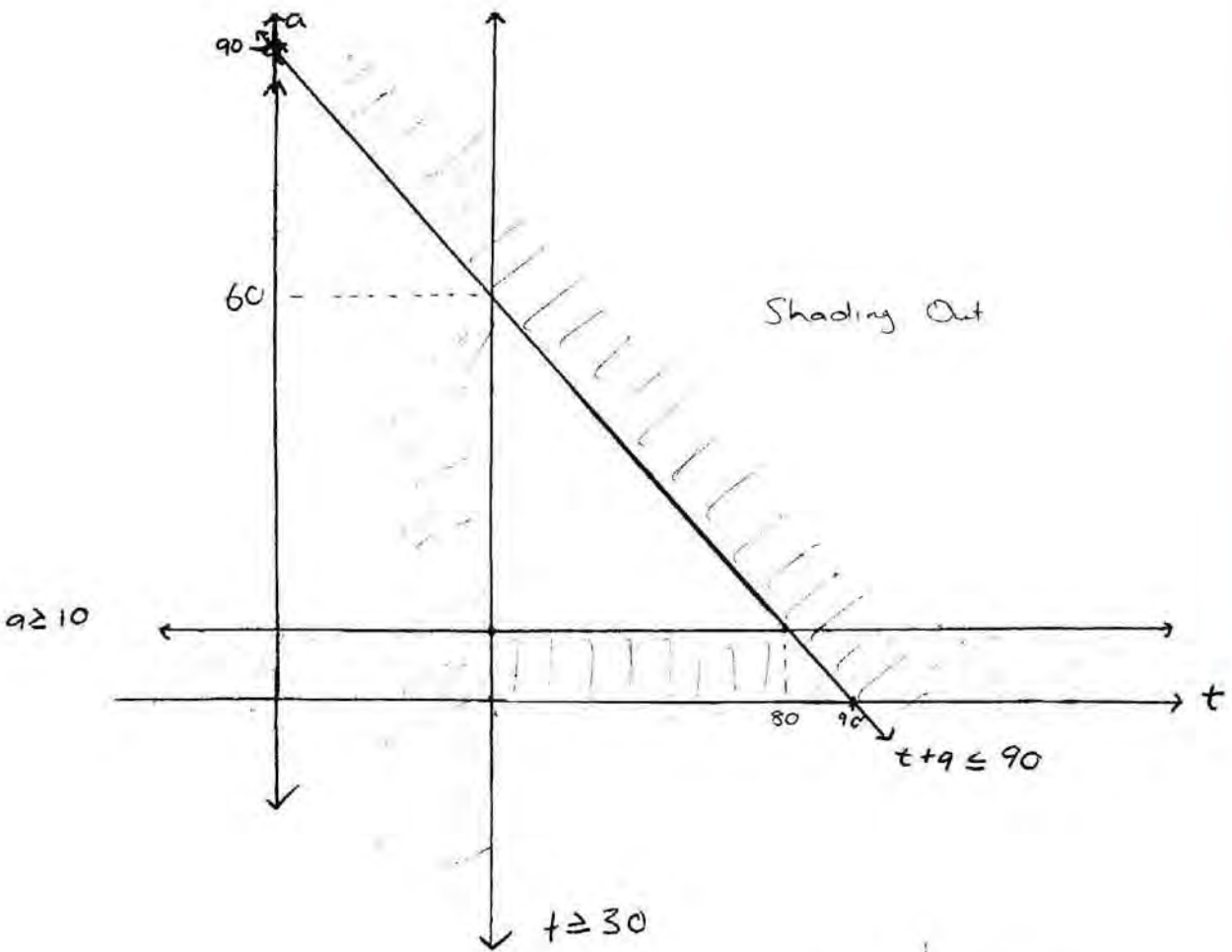
	Grade Boundary: High Not Achieved
6.	For Achieved the student is required to apply linear programming methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations. The student has provided a feasible region showing at least two of the constraints **(1)**. To be awarded Achieved the student would need to form the equation of the inequality for the hours of labour.

Student 6: High Not Achieved

$t + q \leq 90$

$t \geq 30$

$q \geq 10$



1

vertices	income
(30, 10)	
(80, 10)	
(60, 30)	